Terahertz quantum cascade laser frequency combs

David Patrick Burghoff

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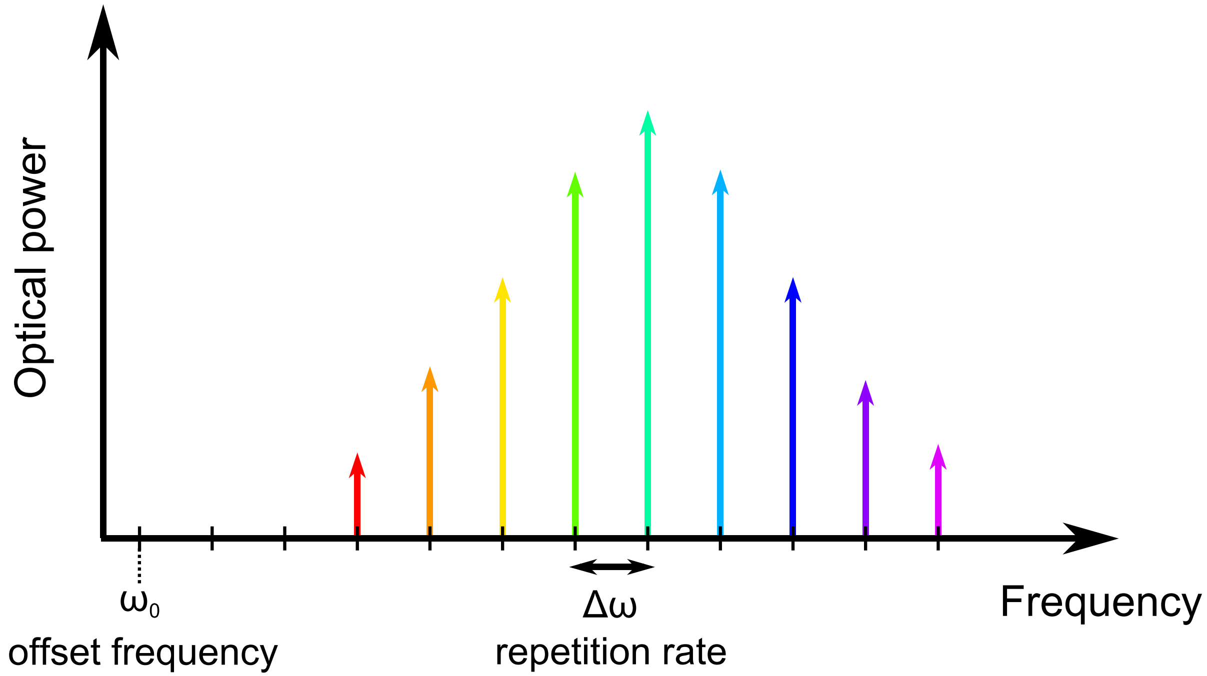
Motivation

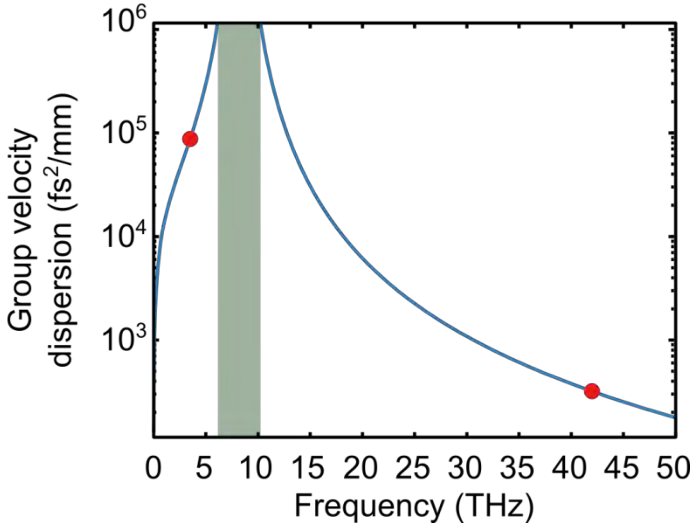
- **Frequency combs**: Light sources that consist of a large number of **evenly-spaced** laser lines

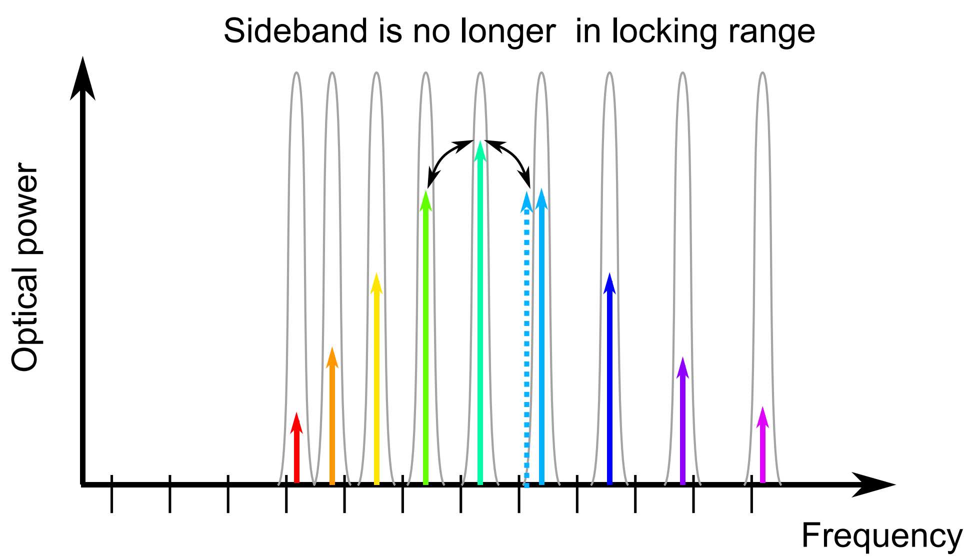


- What strategies can we use to make THz QCL combs?
- Are there any strategies that apply to all mid-IR QCL combs as well?

Key issue: dispersion

- III-V materials are particularly dispersive in THz
  - GaAs at 3.5 THz: 87,400 fs^2/mm
  - Frequencies separated by 1 THz will slip by λ/4 after only 130 μm!

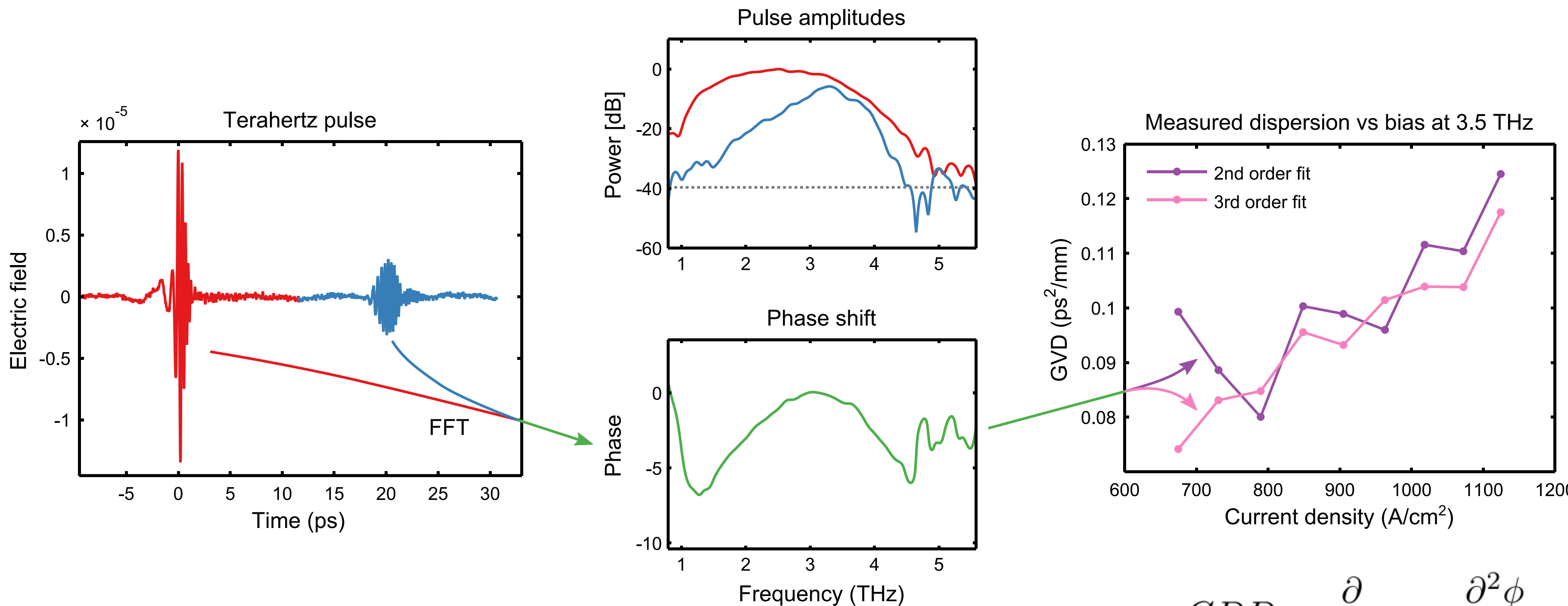




- Injection locking cannot occur when four-wave mixing is too far off-resonance

THz QCL dispersion

- Gain medium actually makes things worse. Can measure real dispersion using THz-TDS (Karl Unterrainer's talk)
- Single section techniques easier for phase measurements



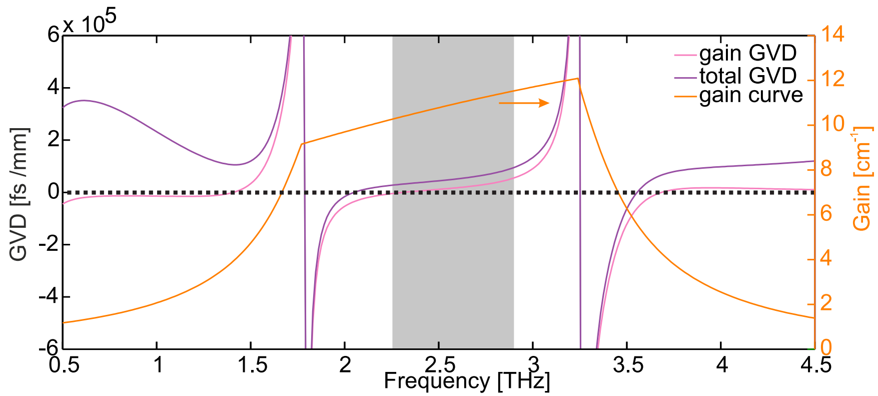
Single-section laser, 774 μm long, 30 μm wide

$$GDD = \frac{\partial}{\partial \omega}\tau_g = \frac{\partial^2 \phi}{\partial \omega^2}$$

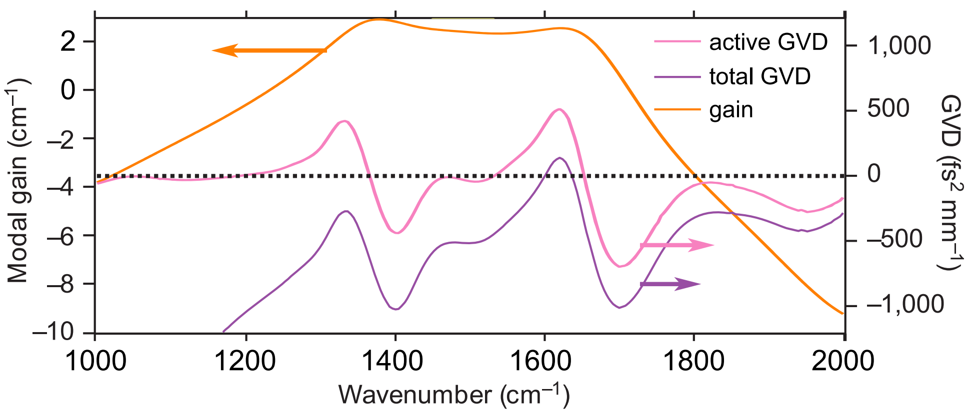
$$GVD = \frac{\partial}{\partial \omega}\frac{1}{v_g} = \frac{\partial^2 k}{\partial \omega^2}$$

Negative GVD from gain medium

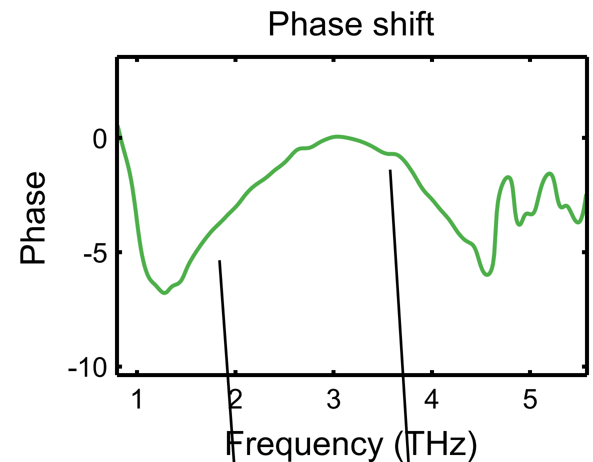
- Broadband gain media often have a region of negative GVD that can partially compensate dispersion



Rosch et al., Arxiv, 2014



Hugi et al., Nature 2012.



Burghoff et al., Nat. Photon. 2014.

region of negative dispersion (concave up)

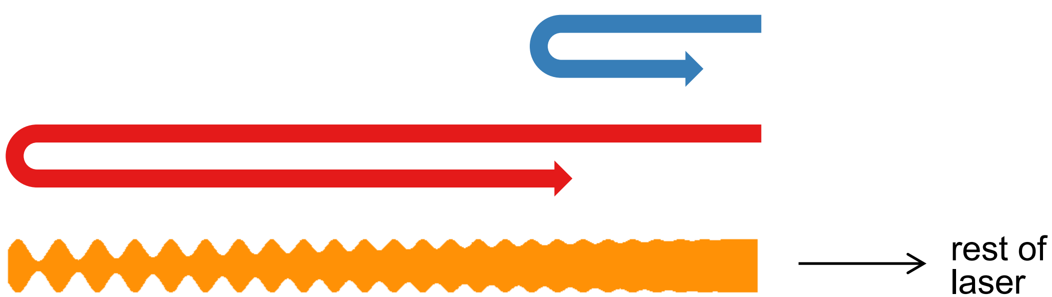
net positive dispersion (concave down)~0.1 ps^2/mm

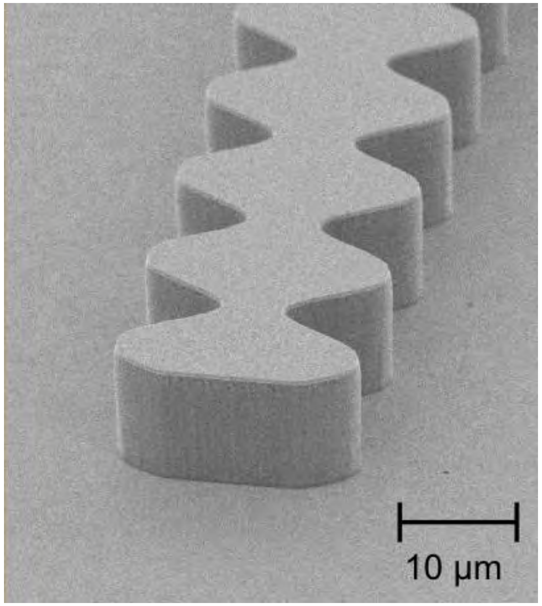
Negative GVD from gain medium

- Broadband gain media often have a region of negative GVD that can partially compensate dispersion
- But without dispersion compensation, combs based on broadband gain media will have difficulty covering more than a fraction of their gain-bandwidth
- Also, many (most?) QCLs don't spontaneously form frequency combs

Dispersion compensation

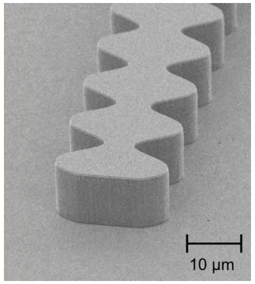
- Need to counteract natural dispersion by delaying long wavelengths relative to short ones
- Double-chirped mirrors (DCMs): a scheme for compensating dispersion traditionally used in ultrashort pulse generation
- Basic idea
  - Chirp the frequency of a DFB (chirp #1)
  - Taper the amplitude (chirp #2)

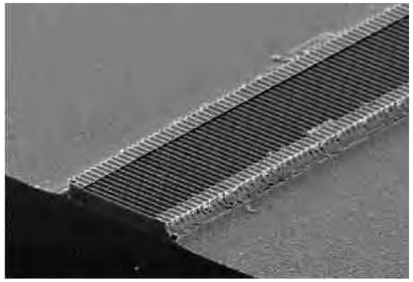
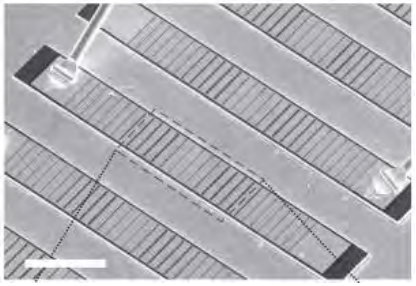
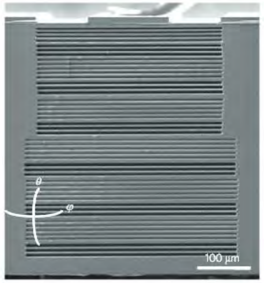




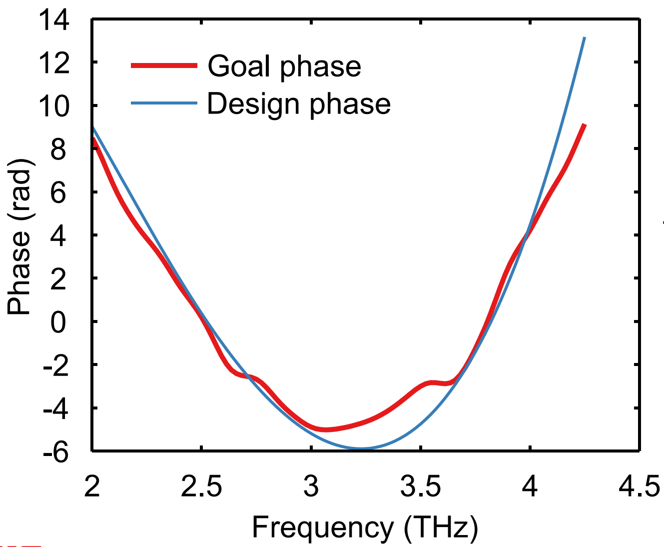
Versatility of DCMs

- Any form of distributed feedback can work…

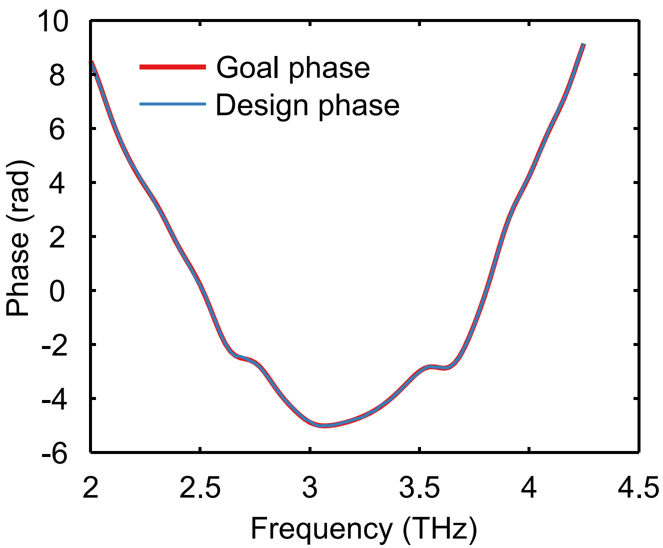


- All sorts of dispersion can be compensated

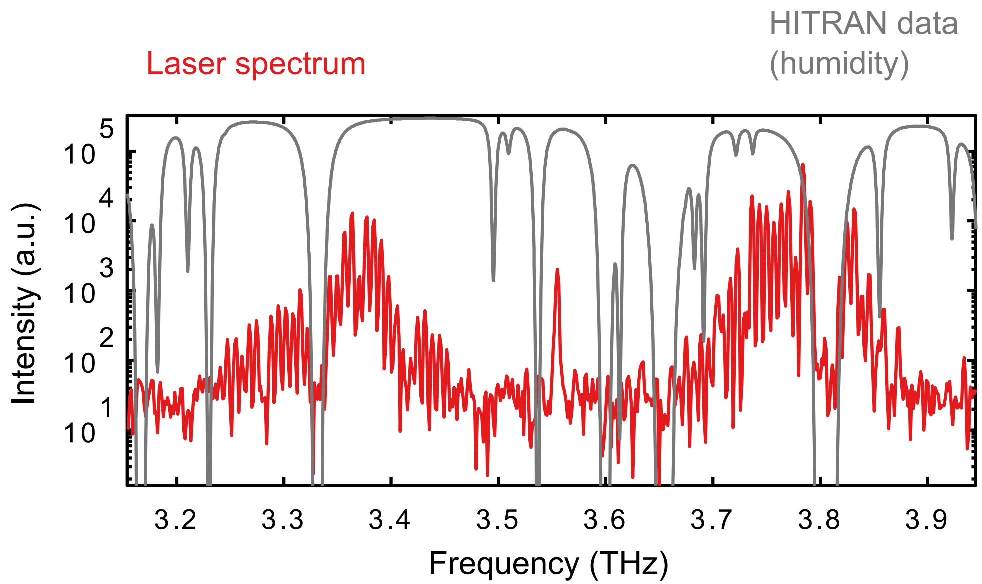



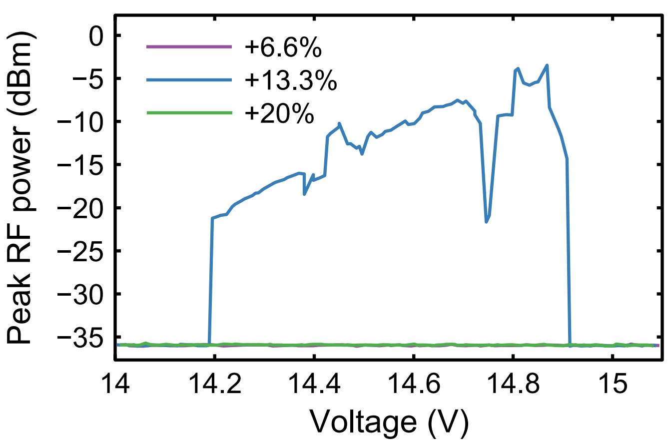
genetic optimization →

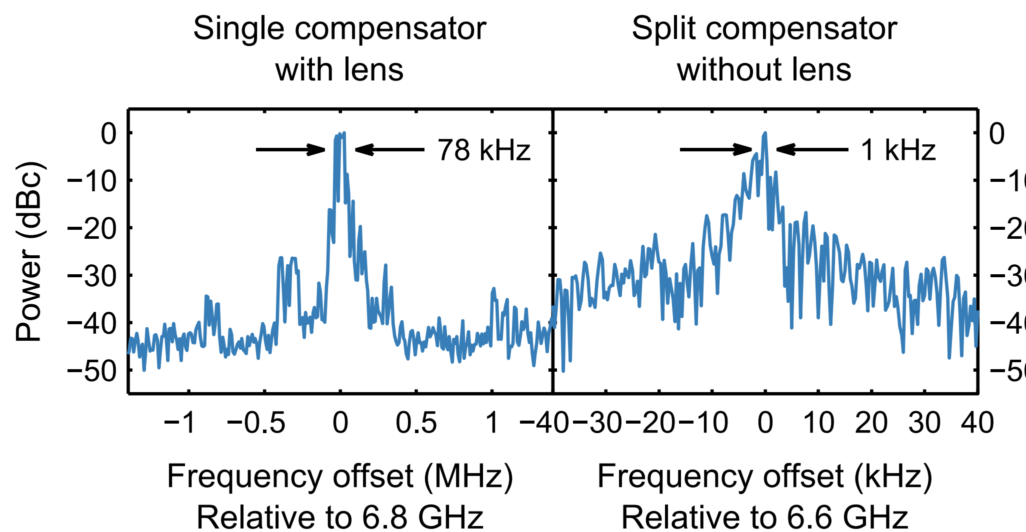


Basic results

- In each dispersion sweep series, one laser produces broad spectrum when DC-biased
- Same device produces strong narrowband RF signal directly from laser at repetition rate (near 6.7 GHz, up to -33 dBm)
  - Very feedback-sensitive

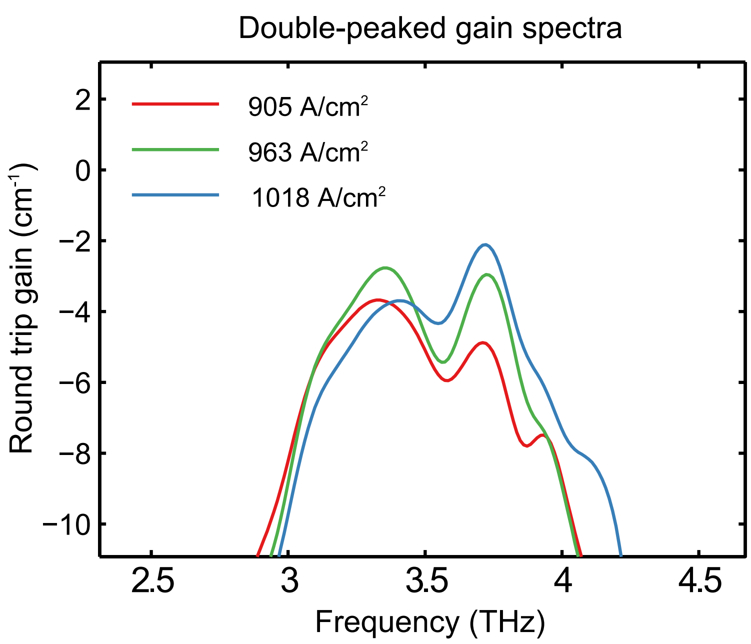


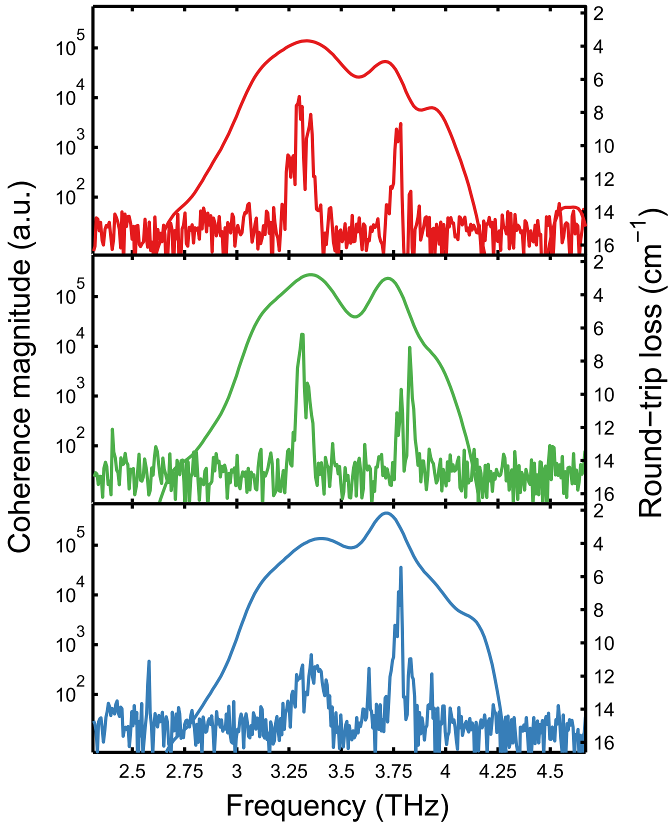




Aside: gain medium

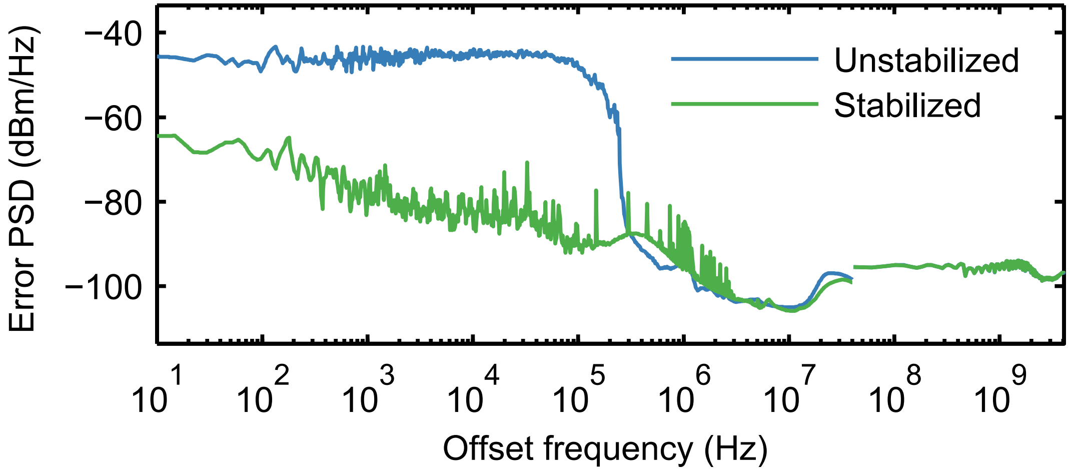
- Splitting of gain spectrum due to gain medium, not coherent instability (Gordon et al., PRA (2008))
  - Gain spectrum splits in a bias-dependent way
  - Large bandwidth (array covers 800 GHz)





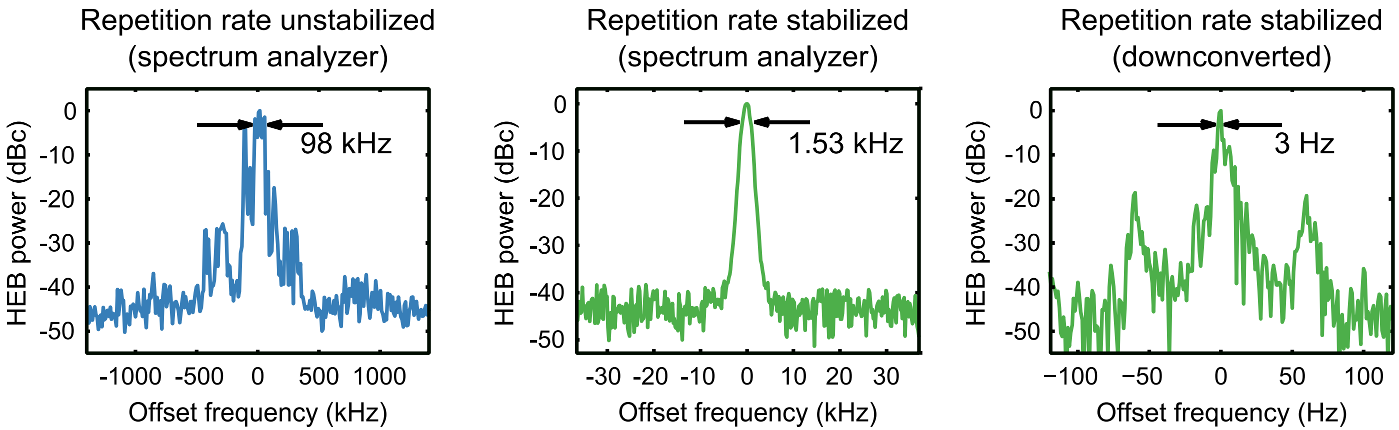
Electronic beatnote

- Electronic beating from laser bias wire is easily stabilized with sub-kHz feedback from PLL by beating it with RF synthesizer



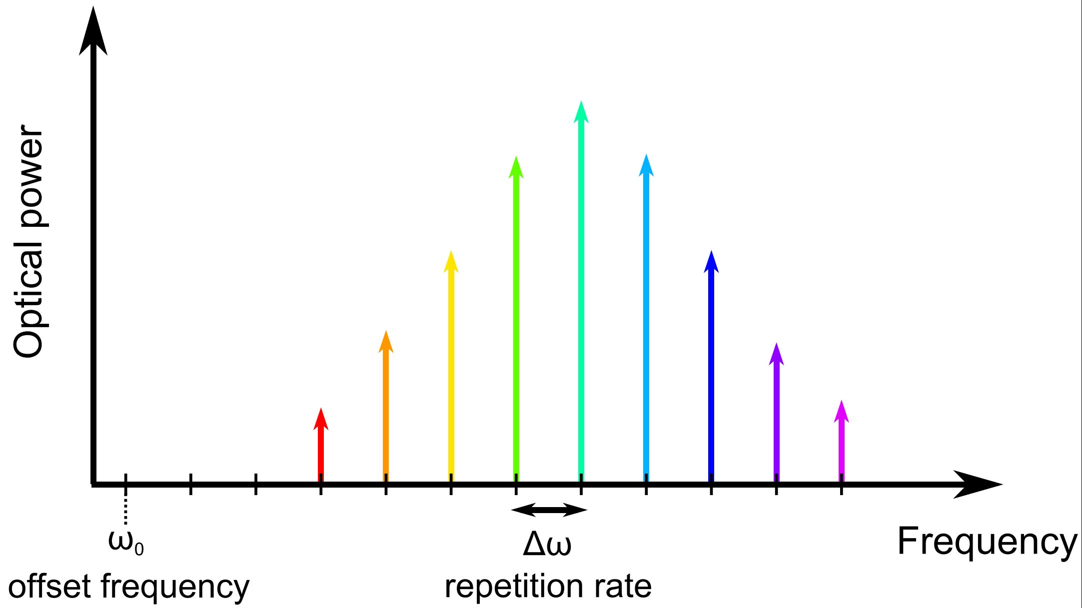
Electronic beatnote is intracavity mixing

- Same beating is observed on fast optical detectors (HEBs and Schottky mixers).



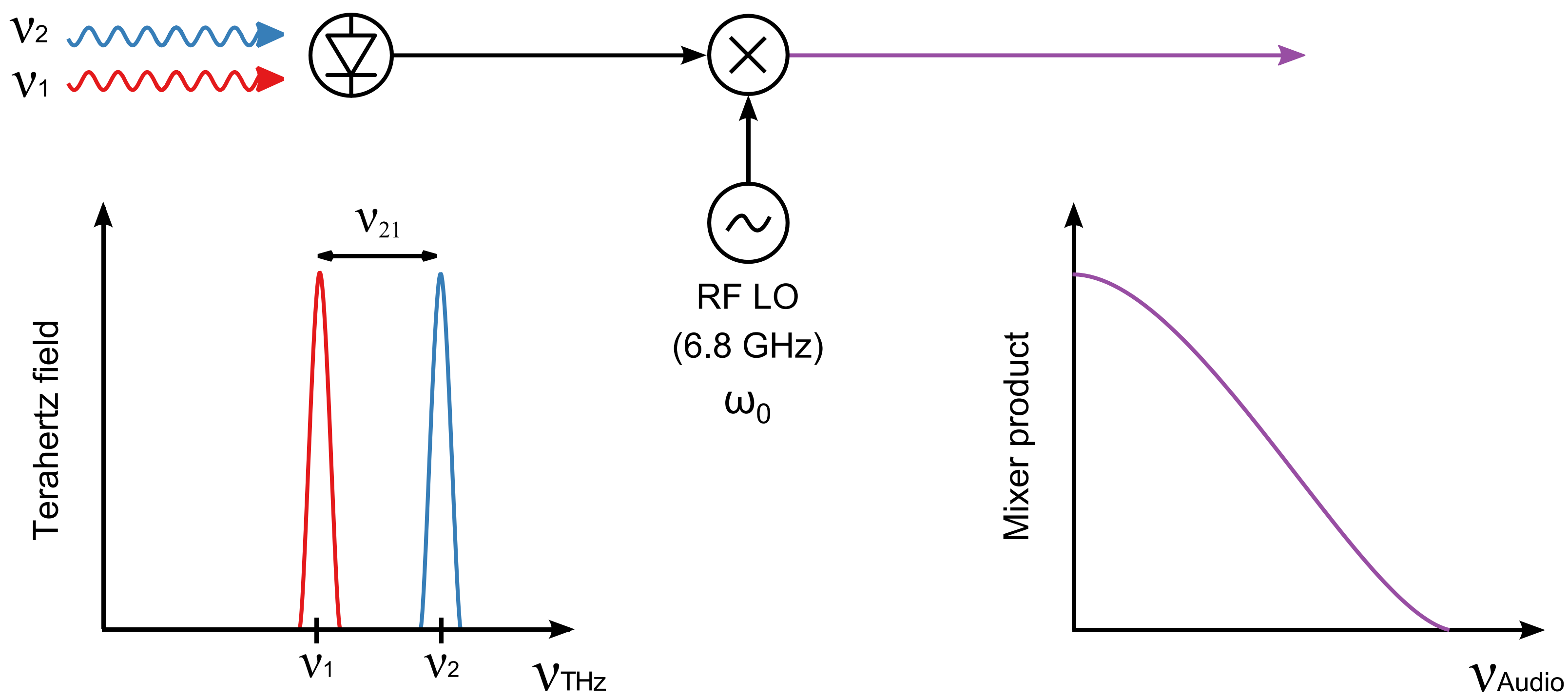
Coherence conditions

- To show that these are actually frequency combs, need to consider two types of coherence:
  - Mutual coherence: are the lines evenly spaced?
  - Absolute coherence: are their linewidths "reasonably" narrow?
- In other words…
  - Mutual coherence: Are the beatnotes all phase-stable (with respect to the repetition rate)?
  - Absolute coherence: Is the offset frequency phase-stable (with respect to a stable clock)?



Mutual coherence of two lines

- Imagine constructing a “coherence detector” for a two-line laser (detector plus downconverter)



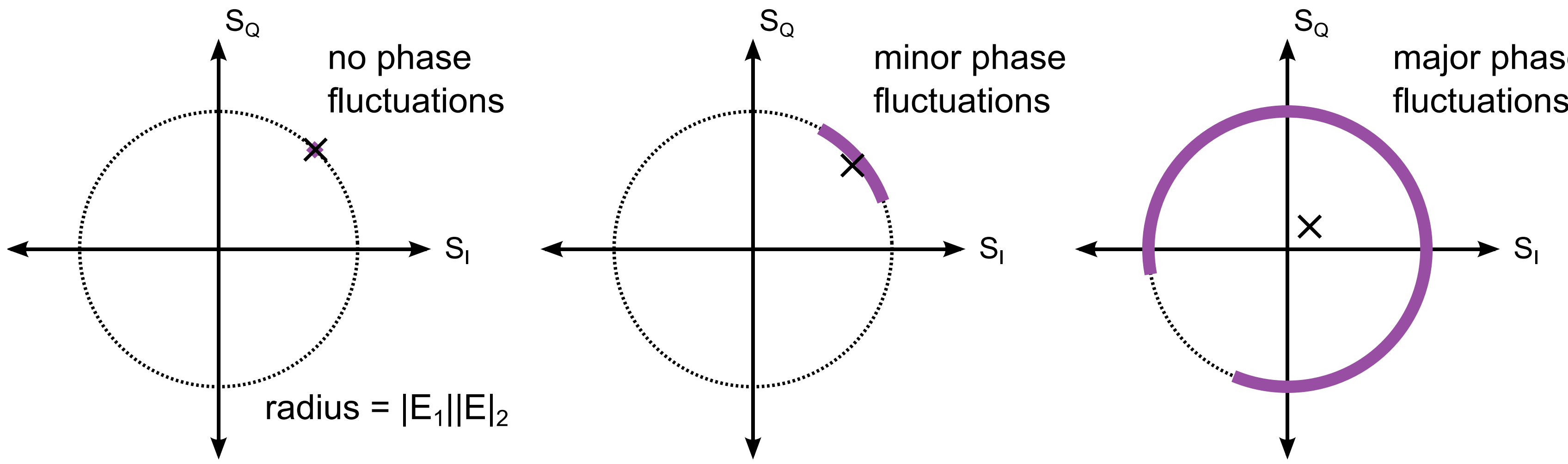
$$E_1(t)e^{i\omega_1 t}$$

$$E_2(t)e^{i\omega_2 t}$$

$$\longrightarrow \quad E_1^*(t)E_2(t)e^{i(\omega_{21}-\omega_0)t} = |E_1 E_2| e^{i\phi_{21}(t)} e^{i(\omega_{21}-\omega_0)t}$$

Mathematical definition of coherence

- Why does $E_1^*(t)E_2(t)e^{i(\omega_{21}-\omega_0)t}$ capture the essence of mutual coherence? Consider the magnitude of its time average:



- Define two-line coherence as:

$$g_{21} \equiv \frac{\left|\left\langle E_1^*(t)E_2(t)e^{i(\omega_{21}-\omega_0)t}\right\rangle\right|}{\sqrt{\langle|E_1(t)|^2\rangle\,\langle|E_2(t)|^2\rangle}}$$

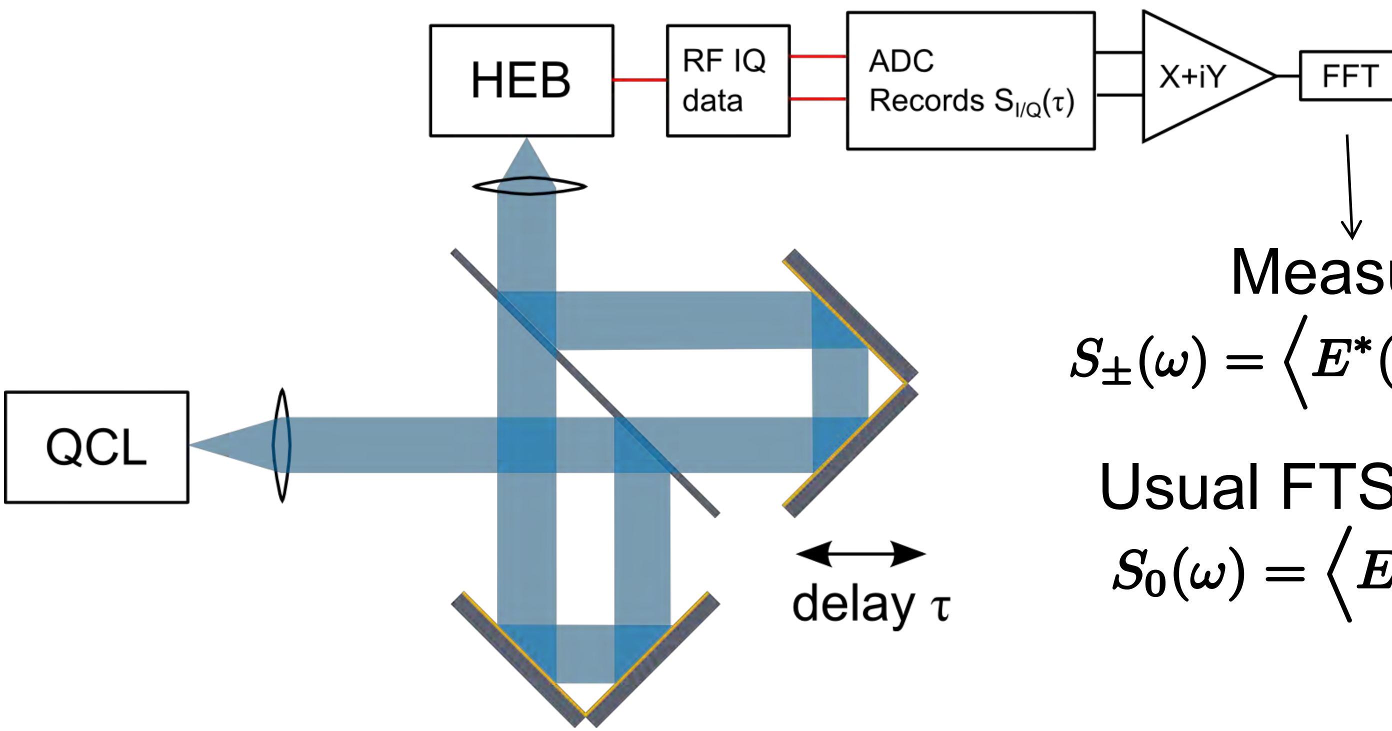
- Generalization to N lines:

$$g_+(\omega) \equiv \frac{|\langle E^*(\omega)E(\omega+\omega_0)\rangle|}{\sqrt{\langle|E(\omega)|^2\rangle\,\langle|E(\omega+\omega_0)|^2\rangle}}$$

Similar definitions in the microcomb literature
Torres-Company et al. *Opt. Express* (2014)

Shifted Wave Interference FTS

- How to measure coherence in the case of N lines? Using our coherence detector alone won't work since all the lines would be measured.
- Instead, do coherent detection of the beatnote at the repetition rate through a Michelson interferometer and FT. We call this **S**hifted **W**ave **I**nterference **FTS = SWIFTS.**
  - Modification of ETH beatnote interferometry, which detects intensity of beatnote vs RF frequency instead



Measures

$$S_{\pm}(\omega) = \left\langle E^{*}(\omega) E(\omega \pm \Delta\omega) \right\rangle$$

Usual FTS measures

$$S_0(\omega) = \left\langle E^{*}(\omega) E(\omega) \right\rangle$$

Density matrix analogy

	Density matrices	Optical coherence
Matrix elements	$\rho_{nm} = \langle c_n c_m^* \rangle$	$S_{nm} = \langle E_n E_m^* \rangle$
Interpretation of on-diagonal elements	Populations $\rho_{nn} = \langle \lvert c_n \rvert^2 \rangle$	Optical power $S_{nn} = \langle \lvert E_n \rvert^2 \rangle$
Interpretation of off-diagonal elements	Coherence	Coherence
Cauchy-Schwarz inequality	$\lvert\rho_{nm}\rvert \leq \sqrt{\rho_{nn}\rho_{mm}}$	$\lvert S_{nm}\rvert \leq \sqrt{S_{nn}S_{mm}}$ $\lvert\langle E_n E_m^* \rangle\rvert \leq \sqrt{\langle \lvert E_n\rvert^2\rangle \langle \lvert E_m\rvert^2\rangle}$

SWIFTS as a way to measure coherence

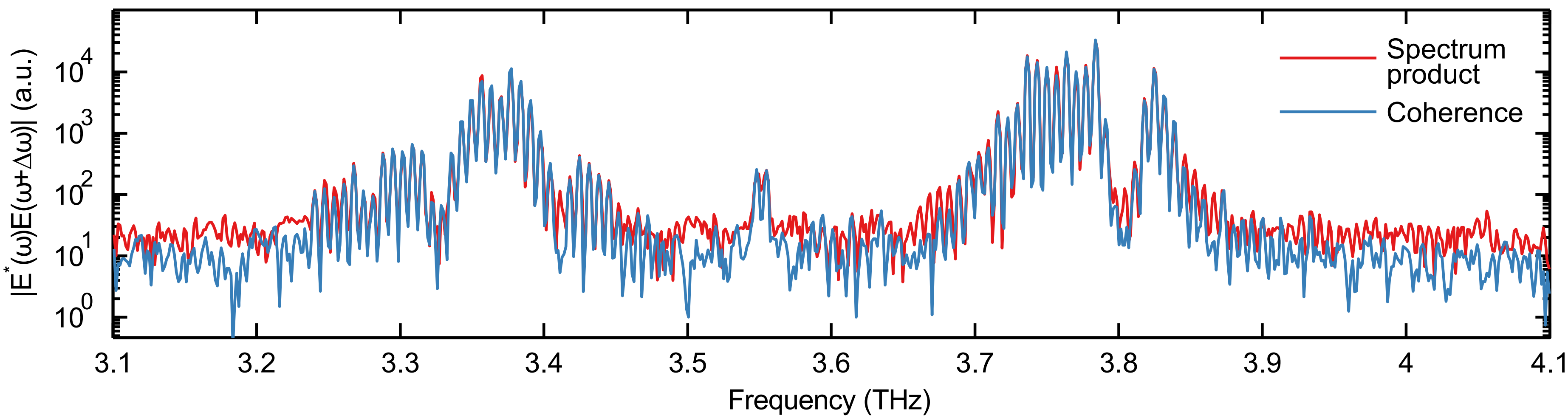
- Normal FTS can be used to measure a spectrum product…

$$S_{\pm}^{sp}(\omega) \equiv \sqrt{\left\langle |E(\omega)|^2 \right\rangle} \sqrt{\left\langle |E(\omega \pm \Delta\omega)|^2 \right\rangle}$$

SWIFTS measures the coherence…

$$S_{\pm}(\omega) = \langle E^*(\omega) E(\omega \pm \Delta\omega) \rangle \qquad g_{\pm}(\omega) \equiv \frac{|\langle E^*(\omega) E(\omega \pm \Delta\omega)\rangle|}{\sqrt{\langle |E(\omega)|^2 \rangle \langle |E(\omega \pm \Delta\omega)|^2 \rangle}}$$

- Equality between spectrum product and correlation is only achieved when all of the modes are completely phase-coherent and spaced *exactly* by the repetition rate (within the lock-in BW, ~Hz)



Almost all of the lines are in the comb!

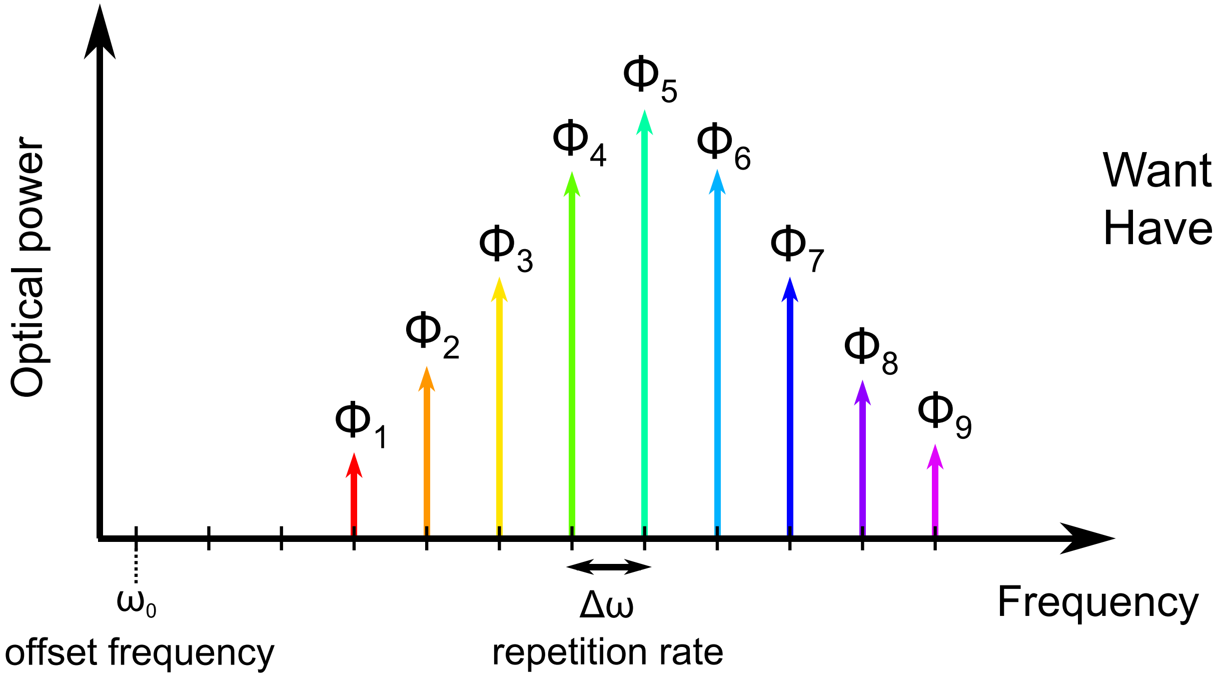
SWIFTS vs Beatnote interferometry

	SWIFTS	Beatnote interferometry
Measurement (τ=delay, ω_0=ref. freq.)	$S_I(\tau,\omega_0) = \left\langle (E(t)+E(t-\tau))^2 \cos(\omega_0 t) \right\rangle$ $S_Q(\tau,\omega_0) = \left\langle (E(t)+E(t-\tau))^2 \sin(\omega_0 t) \right\rangle$	$S_M(\tau,\omega_0) = \sqrt{S_I^2(\tau,\omega_0) + S_Q^2(\tau,\omega_0)}$
Range of ω_0	One frequency at a time (usually repetition rate)	Spectrum analyzer span (usually repetition rate plus some range)
Sensitive to what bandwidth?	Lock-in bandwidth or integration time (Hz-kHz)	Spectrum analyzer resolution bandwidth (Hz to sub-MHz)
Sensitive to incoherent part?	No	Yes
Fourier Transform	$S_\pm(\omega,\omega_0) = \mathscr{F}\left[S_I \pm i S_Q\right](\omega)$ $\sim \langle E^*(\omega) E(\omega+\omega_0) \rangle$	$S_{BI}(\omega,\omega_0) = \mathscr{F}\left[\sqrt{S_I^2 + S_Q^2}\right](\omega)$
How to retrieve optical phase?	Cumulative sum	Phase retrieval (?), followed by cumulative sum

SWIFTS for phase retrieval

- SWIFT spectrum can (almost) be used to completely find E(t) (like FROG or SPIDER)
  - Measures $S_{\pm}(\omega) = \left\langle E^{*}(\omega)E(\omega \pm \Delta\omega)\right\rangle$
  - Contains phase difference of all adjacent modes
  - Could cumulative sum to get comb phases
- Practically, is noise-sensitive.

A swift

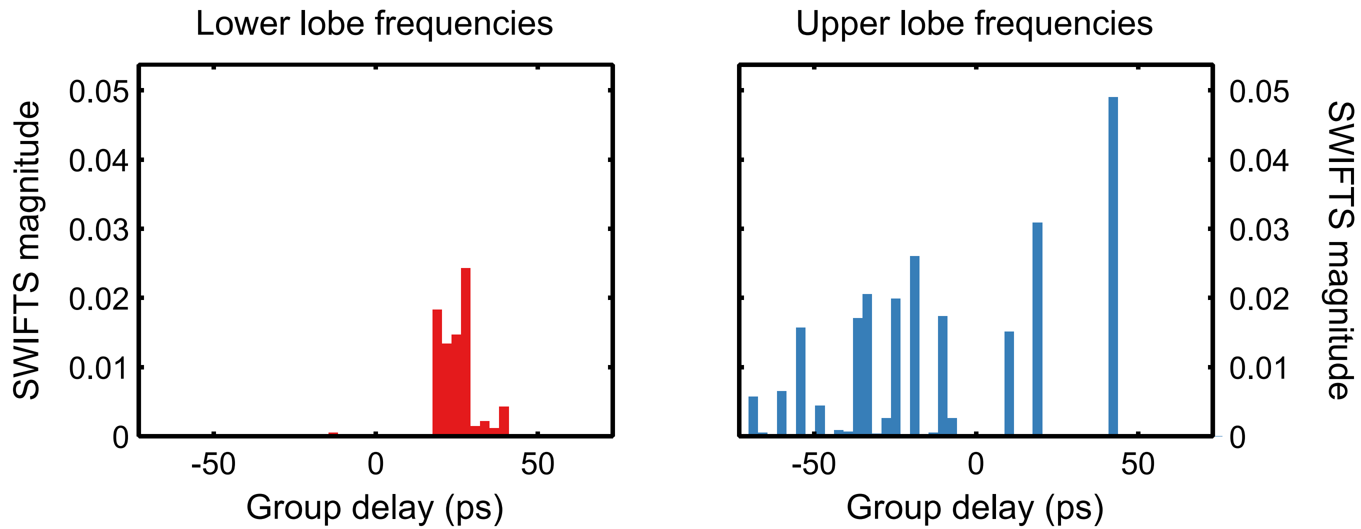


Want Φ_i's
Have Φ_{i+1}-Φ_i

SWIFTS for phase retrieval

- SWIFT spectrum can (almost) be used to completely find E(t) (like FROG or SPIDER)
  - Measures $S_{\pm}(\omega) = \left\langle E^{*}(\omega) E(\omega \pm \Delta\omega) \right\rangle$
  - Contains phase difference of all adjacent modes
  - Could cumulative sum to get comb phases
- Practically, is noise-sensitive.
- Comb is dense, so ΔΦ is approximately the frequency-dependent group delay, given by $\tau_g \approx \Delta\Phi/\Delta\omega$.

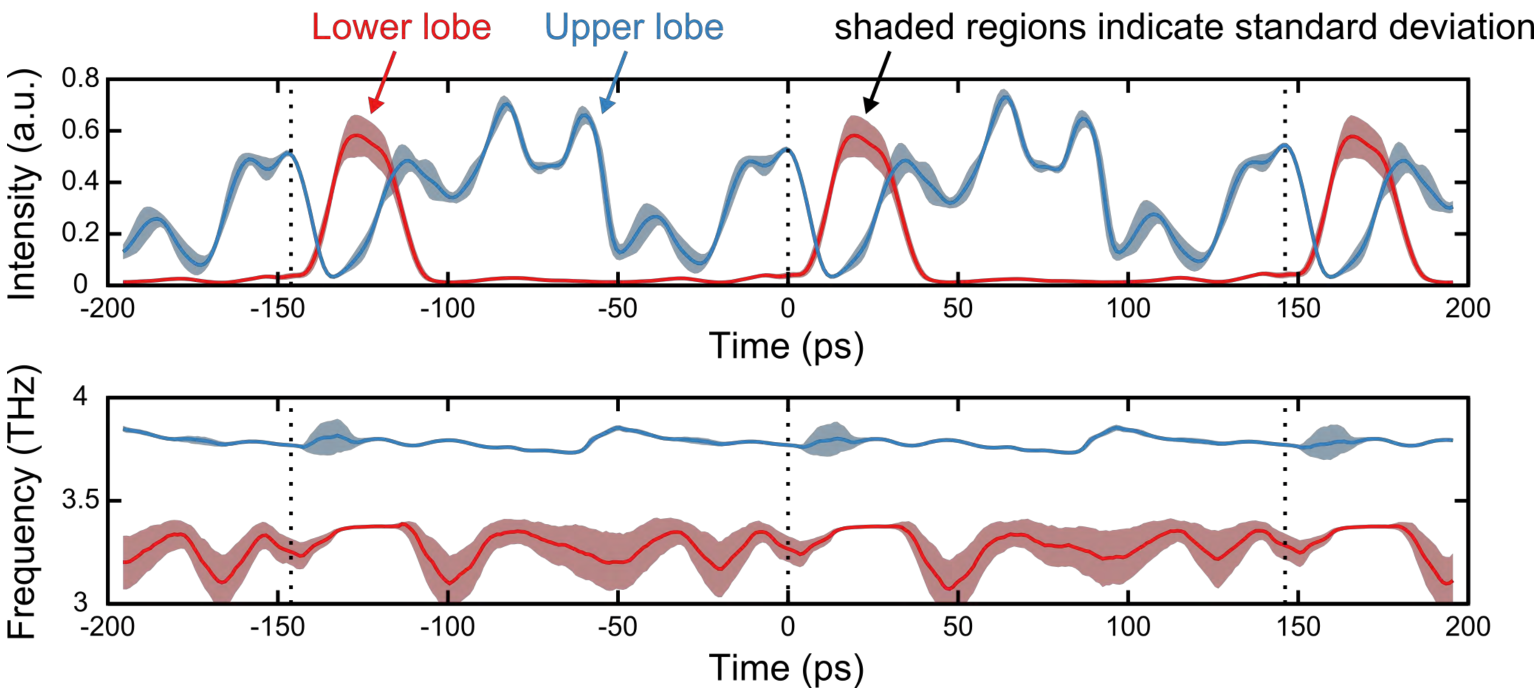
A swift



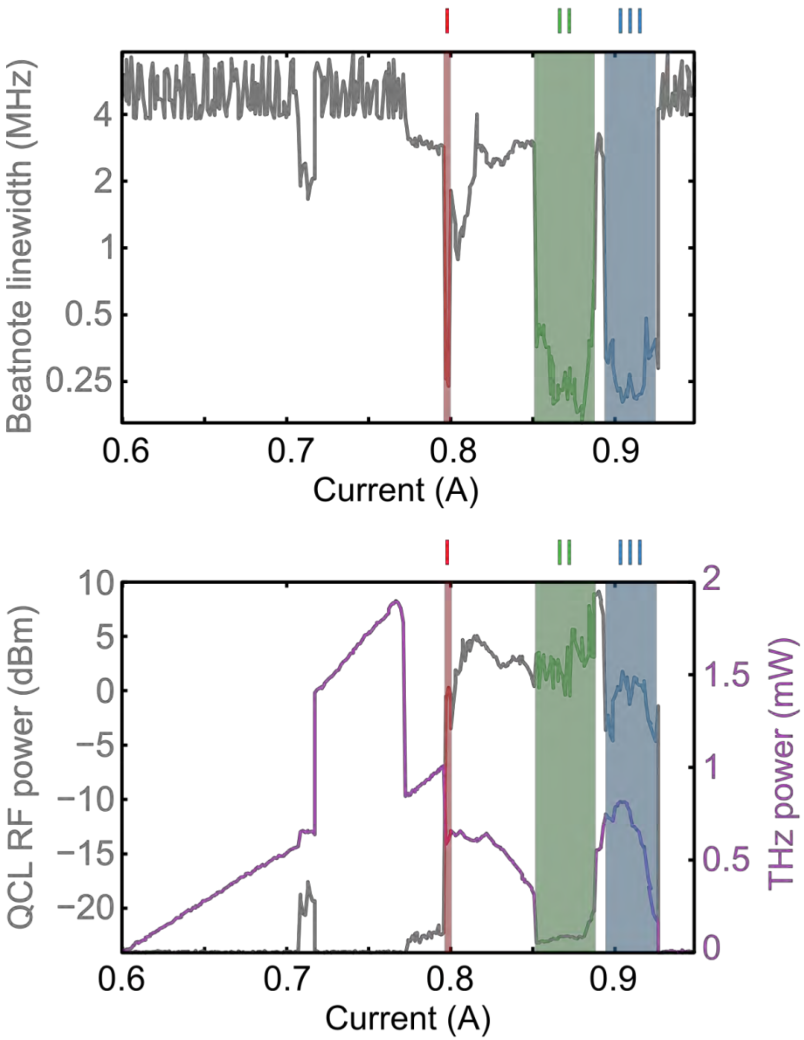
Can we do better?

SWIFTS for time-domain estimation (2)

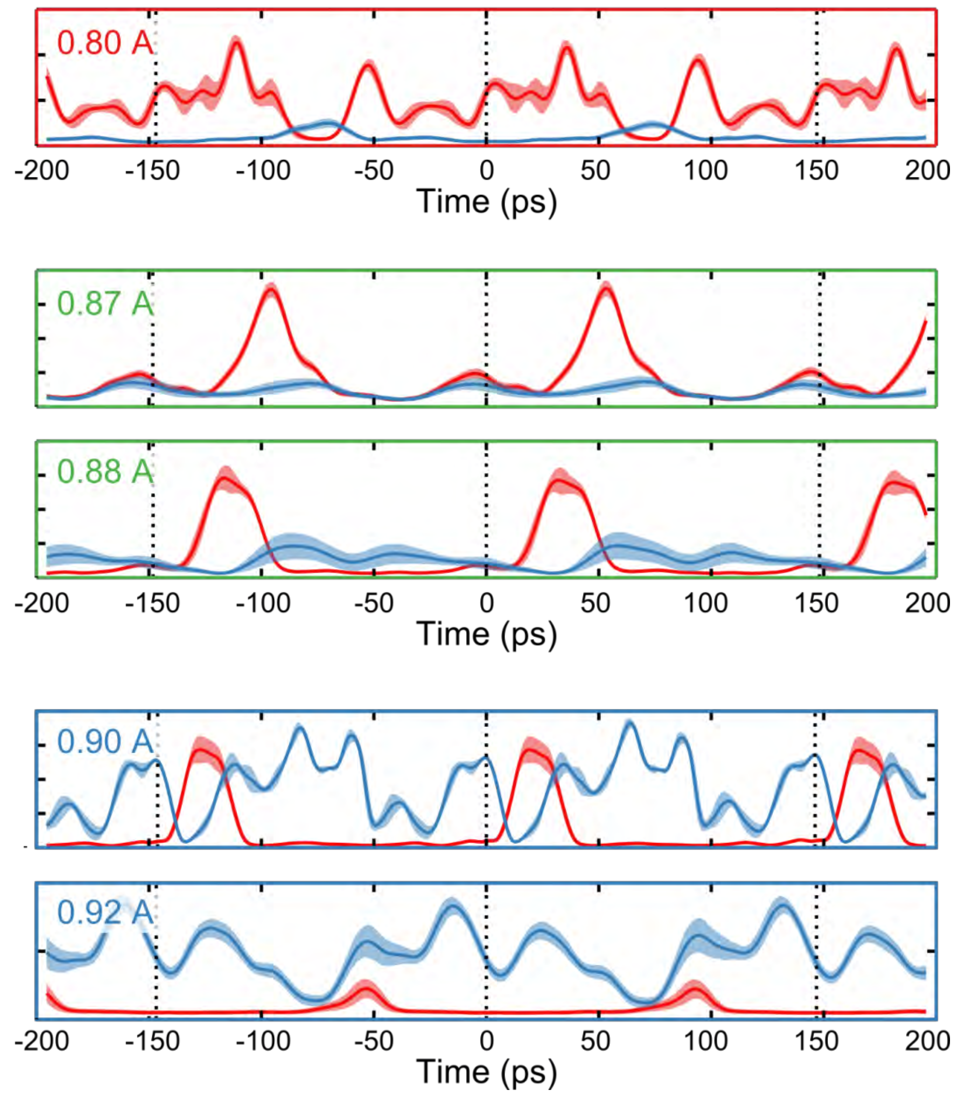
- Basic idea: maximum likelihood estimation
- Noise profile is known; can subtract possible noise values from measurement to get a distribution of “true” values. Then use them to get a distribution of time-domain parameters that are relatively phase-insensitive, like
  - Intensity, I(t)
  - Carrier frequency, f(t)



Intensity versus time versus bias

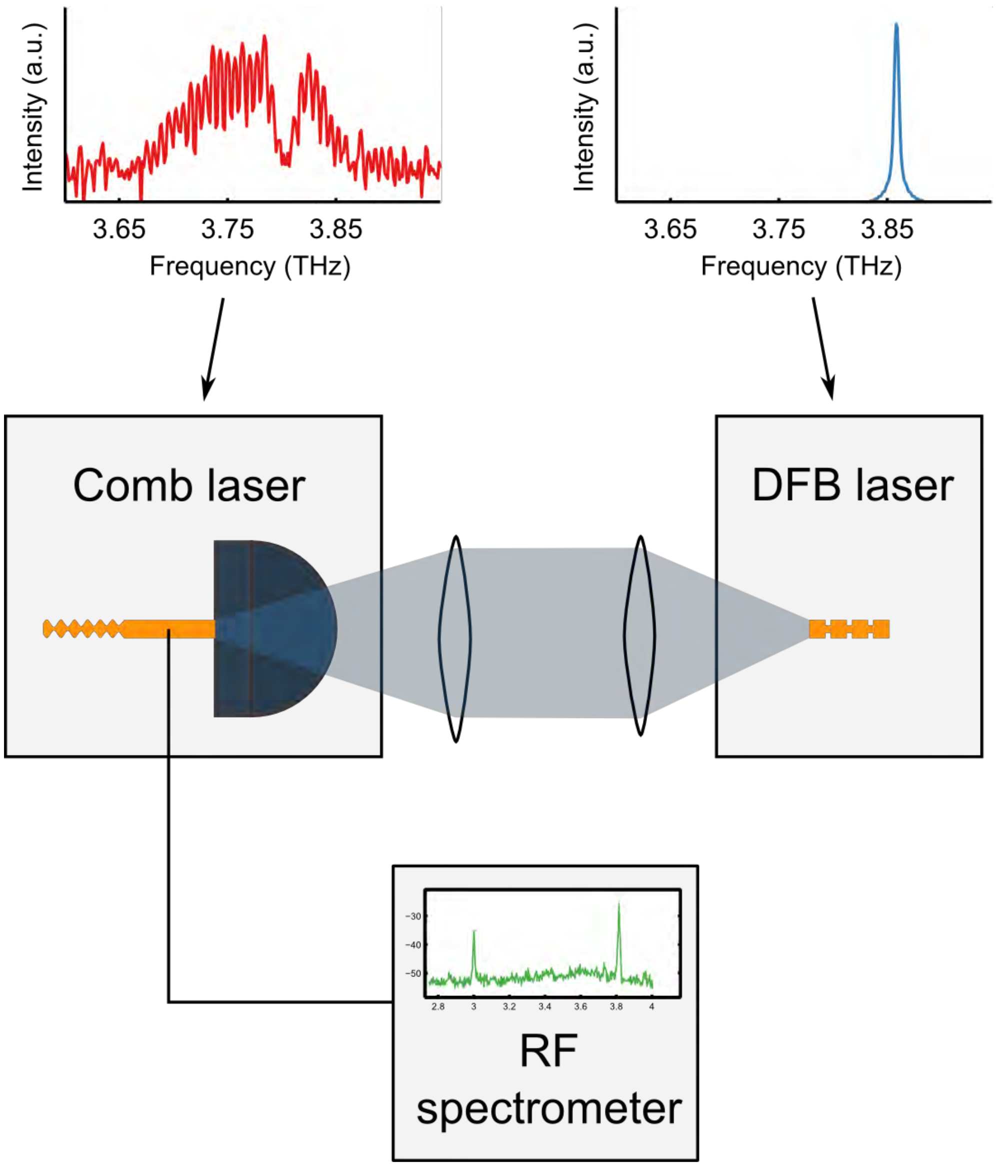


Comb region indicated by bounding box



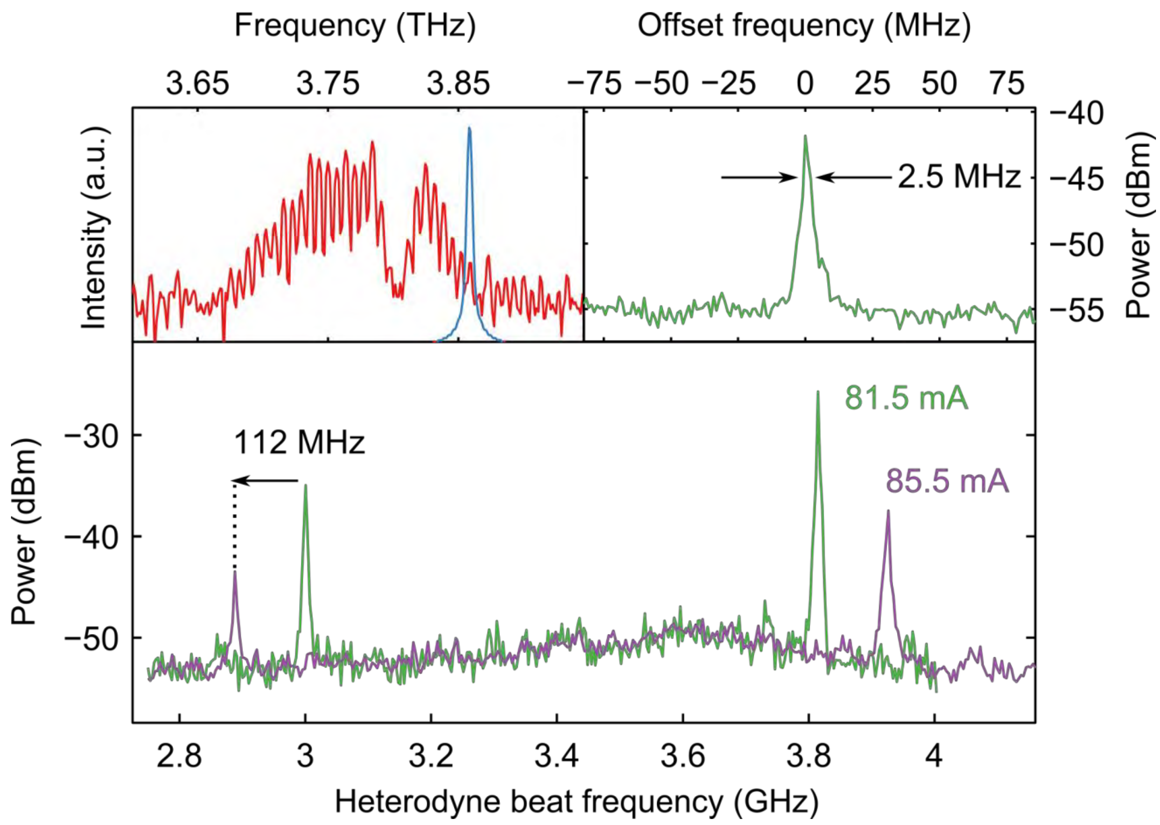
Absolute coherence

- To probe absolute linewidth of comb lines, inject light from a narrowband (DFB) laser into comb cavity and measure intracavity beating between them
- Similar to self-mixing interferometry, only heterodyne
    - Dean et al., *OL* (2011)
    - Talk by J. Keeley on Friday

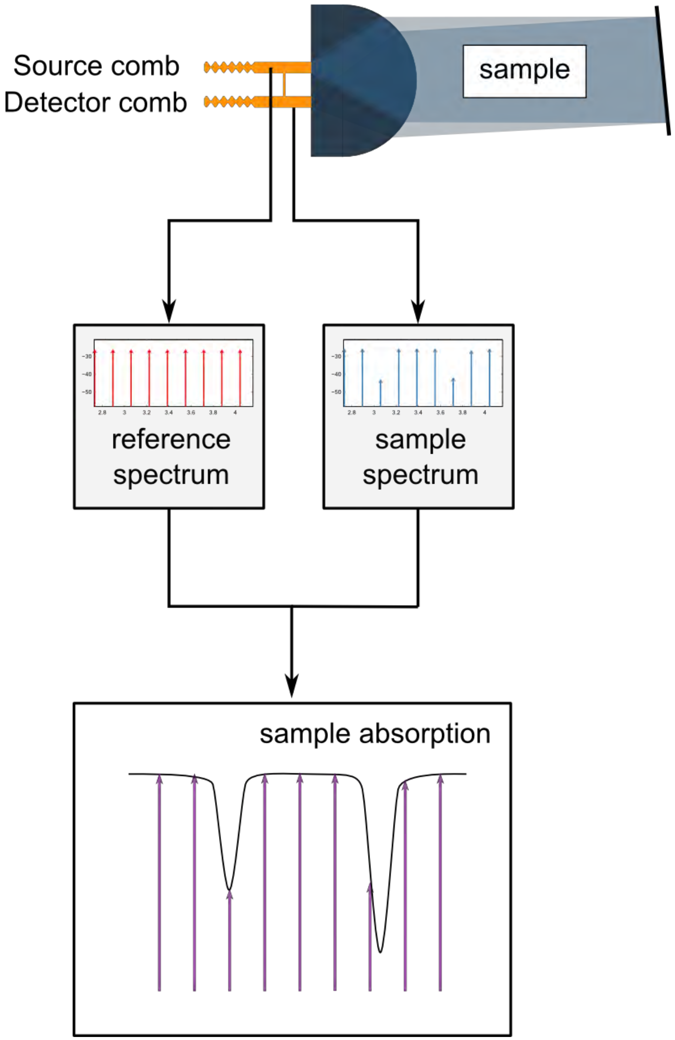


Absolute coherence results

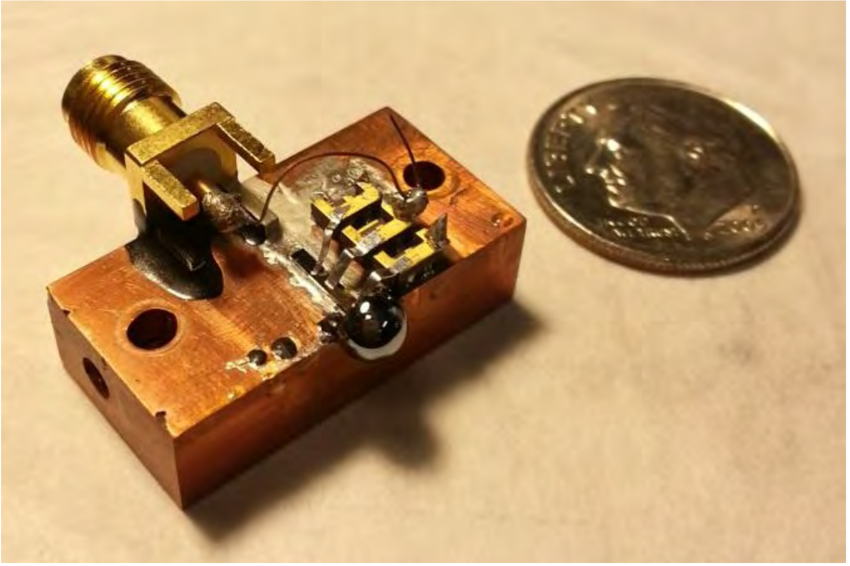
- To probe absolute linewidth of comb lines, inject light from a narrowband (DFB) laser into comb cavity and measure intracavity beating between them
  - Two beatnotes observed that sum to the repetition rate
  - Measured linewidth is 2.5 MHz, deconvolved linewidth is 1.8 MHz (similar to free-running THz QCLs)



Complete solid state terahertz spectrometer on a chip?



- f-2f not that far off (gain medium basically there)
- Intracavity beating for dual comb measurements



Conclusions

- Demonstrated broadband frequency comb generation in THz QCLs using dispersion compensation
  - 500 GHz total coverage (700 GHz with the hole), 70 lines at 50 K
- Developed SWIFTS, an interferometric technique that can be used to measure mutual coherence of a frequency comb and to elucidate its time-domain profile
- Showed that the absolute linewidth of each comb line is comparable to that of typical THz QCLs
  - Possible to use intracavity mixing to make detector-free system?
- See also:
  - Markus Rösch's talk
  - Martin Wienold's poster