Sep.8, 2014

HAMAMATSU
PHOTON IS OUR BUSINESS

Frequency-Noise and Linewidth of Quantum-Cascade Lasers

Masamichi Yamanishi

CRL, Hamamatsu Photonics K.K.

HAMAMATSU PHOTONICS K.K.

e-mail: masamiya@crl.hpk.co.jp

Acknowledgements

Intrinsic Linewidth :
S. Bartalini, S. Borri, and P. De Natale (INO) and Miriam S. Vitiello (CNR)
N. Akikusa (Hamamatsu)

Flicker Noise:
T. Hirohata, S. Hayashi, K. Tanaka and K. Fujita (Hamamatsu)
L. Tombez (UNINE, IBM)

Why frequency-noise and linewidth?

- QCLs: compact sources of MIR and THz radiation for a variety of applications; such as high-precision molecular gas spectroscopy.

- High frequency-stability of the sources is mandatorily demanded.

- The knowledge of frequency-noise properties of QCLs is, nowadays, becoming abundant.

- Review of experimental and theoretical state-of-the art on both of the intrinsic and extrinsic LWs of QCLs running free of any type of feedback effect.

- Focusing on a hot topic, i.e., understanding of electrical flicker-noises and their suppression.

Outline of Presentation

1. Introduction

   Early history of LW study
   Frequency-noise power spectral density

2. Intrinsic linewidth by spontaneous emissions

   The Schawlow-Townes-Henry formula
   Line narrowing by high photon flux at single-mode

3. Line broadening by thermal photons in THz QCLs

   Generalized linewidth formula

4. Flicker frequency and electrical noises

   How to suppress electrical flicker noise
   Hot topic!

5. Summary and future

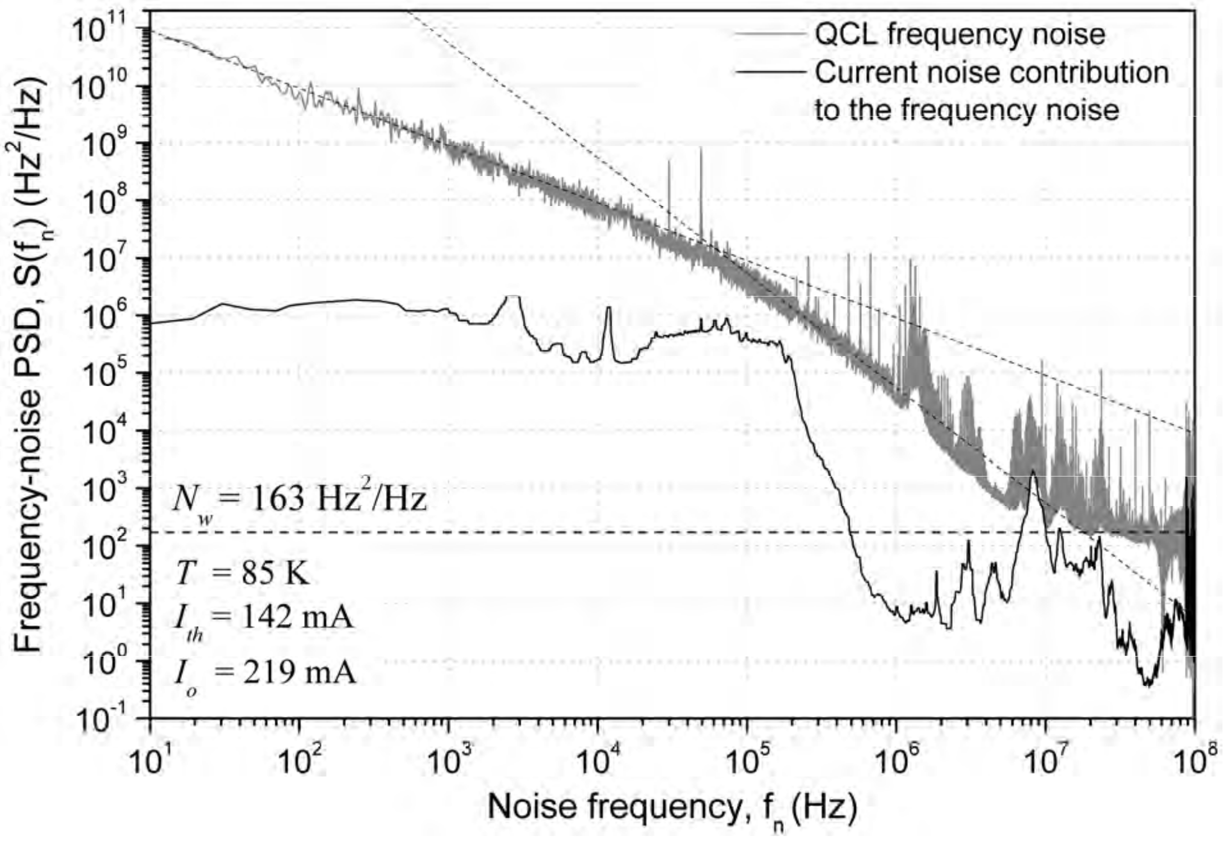
Early history of LW study

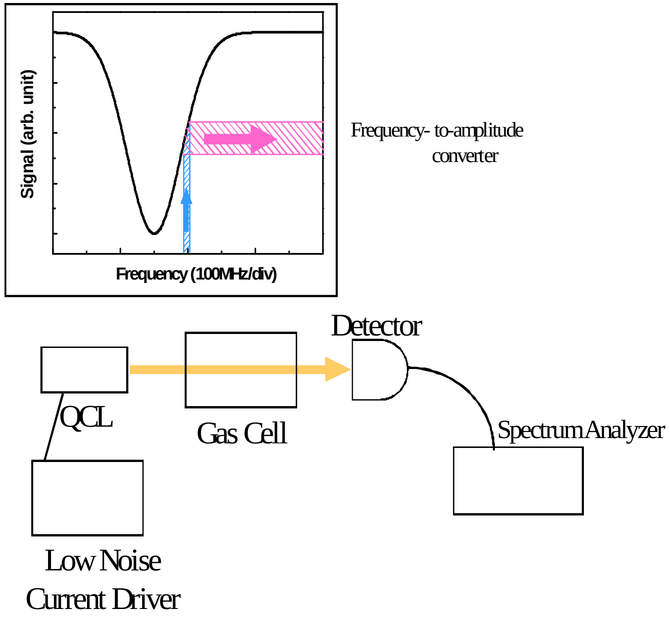
1. J. Faist et al., Superlattices and Microstructures **19,** 337 (1996).
2. S. W. Sharpe et al., Opt. Lett. **23,** 1396 (1998).
3. A. A. Kosterev et al., Opt. Lett. **24,** 1762 (1999).
4. R. M. Williams et al., Opt. Lett. **24**, 1844 (1999). Mid-ir LW~12 kHz by molecular side-locking technique
5. A. A. Kosterev et al., Appl. Opt. **39**, 4425 (2000).
6. H. Ganser et al., Opt. Commun.**197**, 127 (2001).
7. T. L. Myers et al., Opt. Lett. **27**, 170 (2002).
8. D. Weidmann et al., Opt. Lett. **28**, 704 (2003).
9. A. Barkan et al., Opt. Lett. **29**, 575 (2004). THz LW~30 kHz
10. S. Barbieri et al., Opt. Lett. **29**, 1632 (2004). THz LW~20 kHz in multi-mode operation
11. A. L. Betz et al., Opt. Lett. **30**(14), 1837 (2005). THz LW~65 kHz
12. A. Baryshev et al., APL. **89**, 031115 (2006). THz LW~ 6.3 kHz in multi-mode operation

The reported LWs ranged from 100 MHz to a few kHz, affected by environmental effects such as temperature and bias-current fluctuations and by mode-competition noise.

How narrow is the intrinsic LW of a QCL?

13. M. Yamanishi et al., “Theory of the intrinsic linewidth of quantum-cascade lasers: hidden reason for the narrow linewidth and line-broadening by thermal photons,” IEEE J. Quantum Electron. **44**, 12 (2008).
14. S. Bartalini et al., Phys. Rev. Lett. **104**, 083904 (2010). Intrinsic LW~510 Hz of a DFB QCL at 90 K Frequency-noise PSD measurement and a low noise current driver.





Frequency-Noise Power Spectral Density (FN PSD) Measurement;

Intrinsic LW～510 Hz=πN_w

S. Bartalini et al., PRL **104**, 083904 (2010).

Frequency-noise power spectral density (PSD)

Gianni Di Domenico, et al., Applied Optics **49**, 4801 (2010).

The knowledge of the frequency-noise PSD provides us with much more information on the laser noise. The ensemble-averaged autocorrelation function $\Gamma_E(\tau)=\langle E^*(t)E(t+\tau)\rangle$ of the laser field $E(t)=E_0\exp[i(2\pi\nu_0 t+\phi(t))]$ is related to the frequency-noise PSD $S_{\delta\nu}(f_N)$ as a function of the noise Fourier-frequency f_N

$$\Gamma_E(\tau)=E_0^{\,2}\exp[i2\pi\nu_0\tau]\exp[-2\int_0^\infty S_{\delta\nu}(f_N)\frac{\sin^2(\pi f_N\tau)}{f_N^{\,2}}\mathrm{d}f_N]. \qquad (1)$$

One obtains the **laser emission line shape** from the autocorrelation function $\Gamma_E(t)$ by using the Wiener-Khinchin theorem,

$$\mathrm{L}_E(\delta\nu)=4\,\mathrm{Re}\left(\int_0^\infty \exp[-i2\pi\nu\tau]\Gamma_E(\tau)\mathrm{d}\tau\right), \qquad (2)$$

where $\delta\nu=\nu-\nu_0$ is the laser frequency deviation.

The white frequency-noise PSD, $S_{\delta\nu}(f_N)=N_{\mathrm{white}}$ leads to Lorentzian line shape with a FWHM equal to πN_{white}.

Approximate formulas for line shapes in flicker noise cases are given.

Outline of Presentation

1. Introduction

Early history of LW study
Frequency-noise power spectral density

2. Intrinsic linewidth by spontaneous emissions

The Schawlow-Townes-Henry formula
Line narrowing by high photon flux at single-mode

3. Line broadening by thermal photons in THz QCLs

Generalized linewidth formula

4. Flicker frequency and electrical noises

How to suppress electrical flicker noise
Hot topic!

5. Summary and future

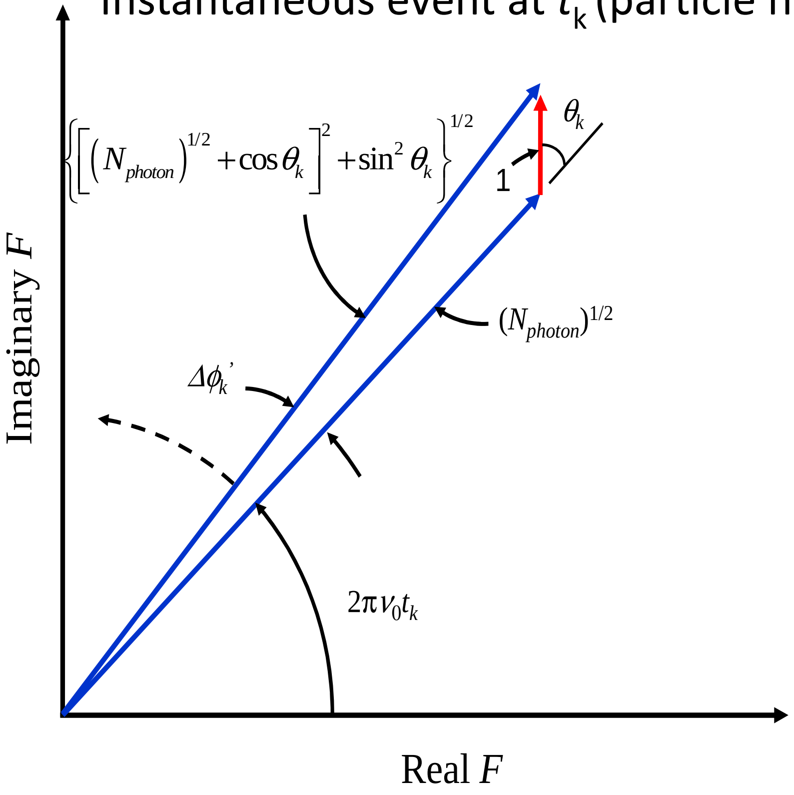
The Schawlow-Townes-Henry formula (I)

Following an intuitive approach by C. H. Henry.

The coherent (classical) laser field: a “phaser” vector rotating with an angular frequency $2\pi\nu_0$,

$$F = (N_{\text{photon}})^{1/2} \exp(i2\pi\nu_0 t). \qquad (1)$$ (for normalized field)

Wave-particle dual nature of a photon by spontaneous emission:
Instantaneous event at t_k (particle nature) with phase angle θ_k (wave nature)



The **direct** instantaneous change $\Delta\phi'_k$ in the phase: for $N_{\text{photon}} \gg 1$,

$$\Delta\phi'_k = \sin\theta_k / (N_{\text{photon}})^{1/2}. \qquad (2)$$

The second contribution to the phase change $\Delta\phi''_k$ induced by the intensity change:

$$\Delta\phi''_k = -\alpha_\text{c} \cos\theta_k / (N_{\text{photon}})^{1/2}. \qquad (3)$$

where the intensity change,

$$\Delta N_{\text{photon},k} = 1 + 2(N_{\text{photon}})^{1/2}\cos\theta_k \sim 2(N_{\text{photon}})^{1/2}\cos\theta_k.$$

The Schawlow-Townes-Henry formula (II)

The total phase fluctuation induced by $T=[M(\beta/\tau_r)N_3]\,t-$ emission events is given by

$$\Delta\phi=\sum_{k=1}^{T}[\Delta\phi'_k+\Delta\phi''_k]=\sum_{k=1}^{T}(1/N_{\text{photon}})^{1/2}[\sin\theta_k-\alpha_c\cos\theta_k]. \qquad (4)$$

Each square-phase-fluctuation term $<[\sin\theta_k-\alpha_c\cos\theta_k]^2>/N_{\text{photon}}=(1/2)(1+\alpha_c^2)/N_{\text{photon}}$ **additively** contributes to the square-phase noise $<(\Delta\phi)^2>$ since spontaneous emission process is **random stochastic one** (Poisson point process).

The average square-phase-fluctuation:

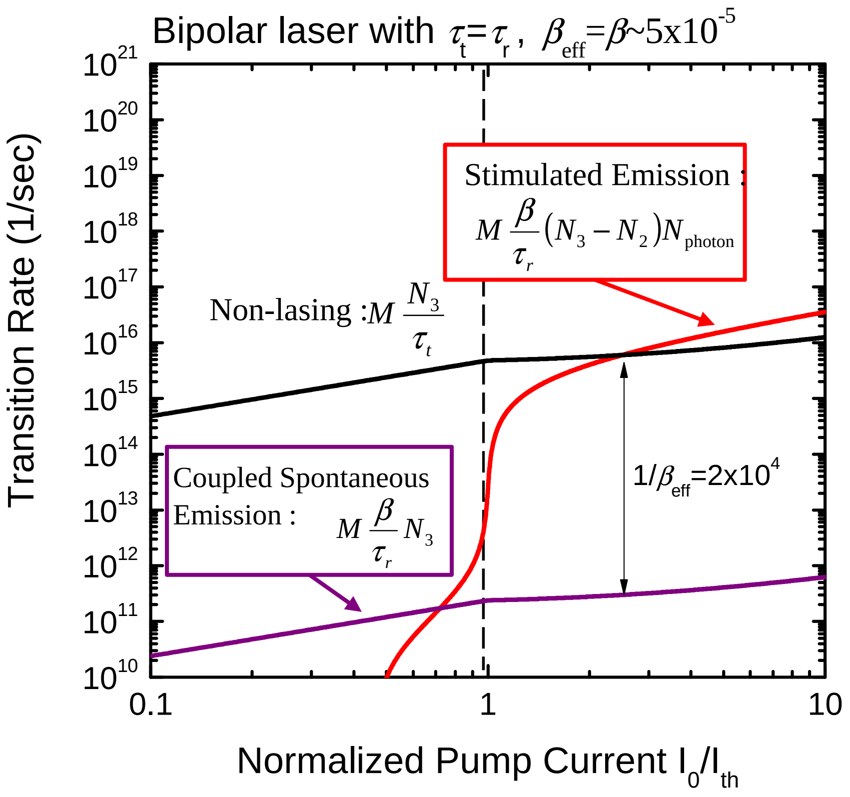
$$<(\Delta\phi)^2>=(1+\alpha_c^{\,2})[M(\beta/\tau_r)N_3]t/2N_{\text{photon}}. \qquad (5)$$

The line shape of the laser output given by the Fourier transform of $<\mathrm{F}(t)^*\mathrm{F}(0)>=|\mathrm{F}(0)|^2\exp(-<(\Delta\phi)^2>/2)$: Lorentzian with an FWHM of

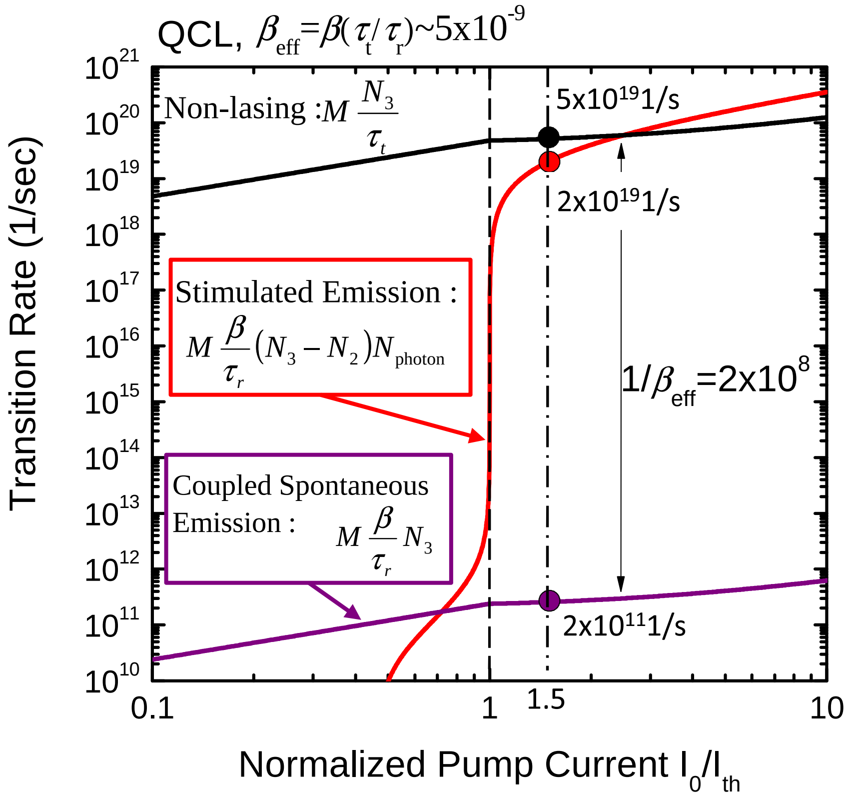
$$\delta\nu=(1/4\pi)(1+\alpha_c^{\,2})[M(\beta/\tau_r)N_3]/N_{\text{photon}}. \qquad (6)$$

The Schawlow-Townes-Henry (STH) formula is safely applicable to a mid-infrared QCL because of the mode-stabilization without relaxation oscillation due to the very fast nonradiative relaxation for upper-level electrons.

The Lorentzian line shape comes out from white (shot) noise in the frequency-noise PSD.



(a) Bipolar laser



(b) QCL

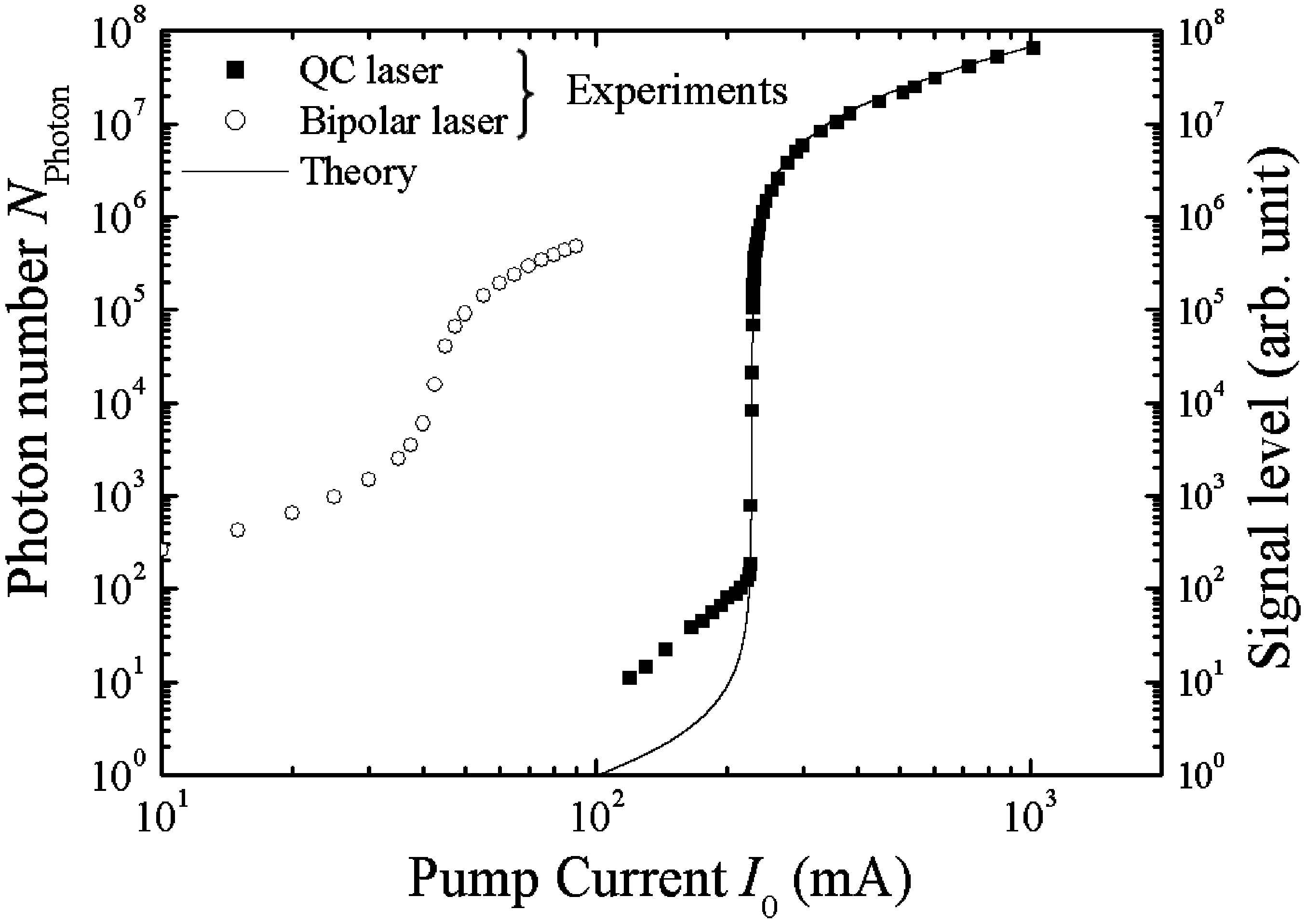
Narrow intrinsic LW: Stable single-mode operation at high photon flux level

$$M(\beta/\tau_r)N_3 << M(\beta/\tau_r)(N_3-N_2)N_{photon} < MN_3/\tau_t$$

Line narrowing

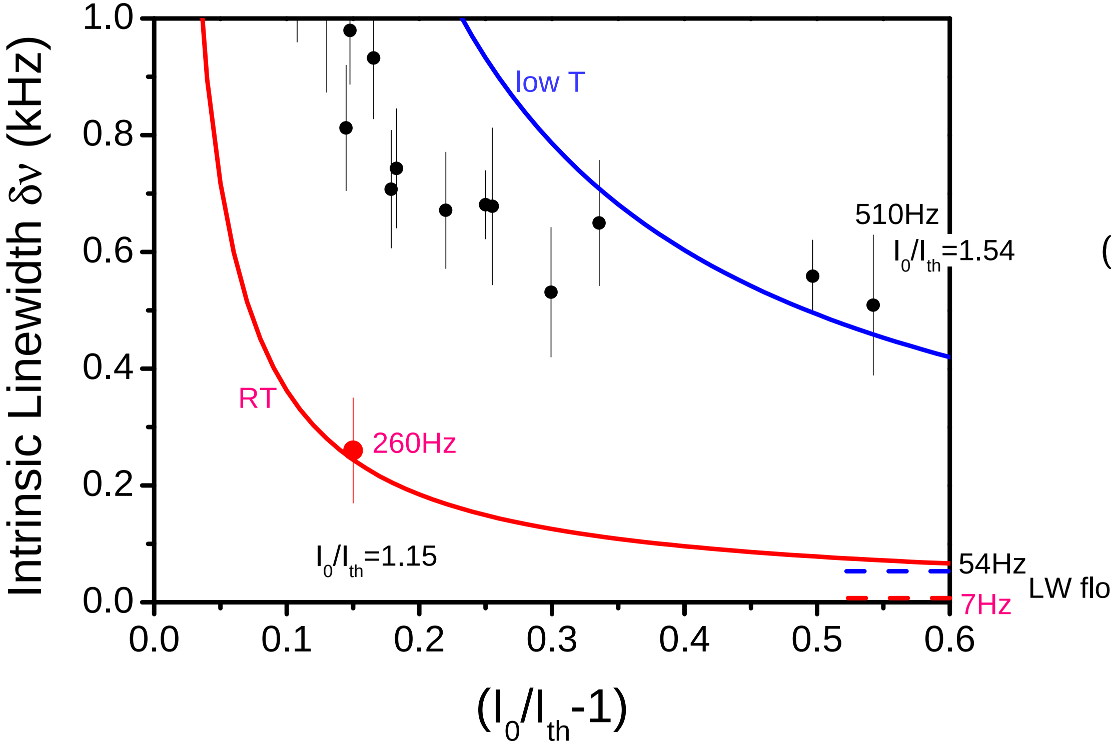
Stable single-mode operation

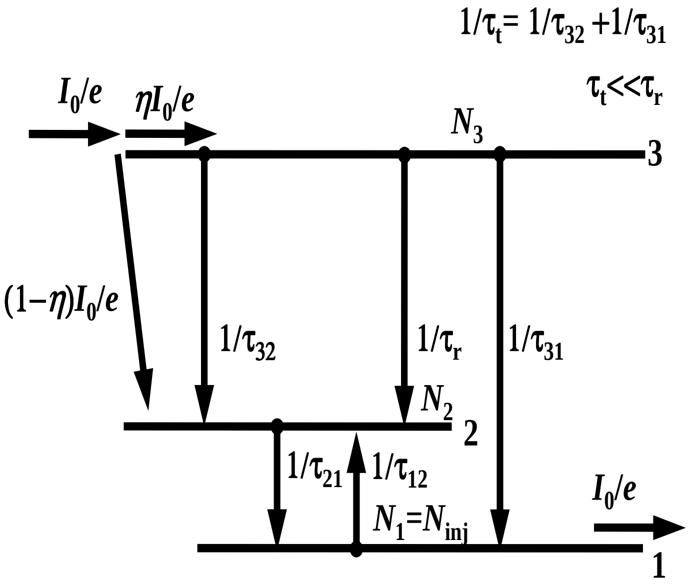
How high photon population N_{photon} is possible in stable single-mode operation?



QC laser
Bipolar laser
Experiments
Theory
Photon number N_{Photon}
Signal level (arb. unit)
Pump Current I_0 (mA)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^1
10^2
10^3

S. Bartalini et al., Optics Express **19**, 17996 (2011). (in collaboration between INO and HPK); $\delta\nu$~260 Hz at RT





$$\delta\nu = \frac{(1+\alpha_c{}^2)M(\beta/\tau_r)N_3}{4\pi N_{\text{photon}}}$$

$$= \frac{(1+\alpha_c{}^2)n_{sp0}\gamma}{4\pi}\left[\frac{\beta_{\text{eff}}(1+\tau_{21}/\tau_{31})}{I_0/I_{\text{th}}-1}+\frac{(\tau_{21}\beta/\tau_r)}{\eta}\right].$$

$\beta_{\text{eff}}=\tau_t\beta/\tau_r \sim 10^{-8}$–$10^{-9}$

$\tau_t << \tau_r$

M. Yamanishi et al. IEEE J-Quantum Electron. **44**, 12 (2008).

Outline of Presentation

1. Introduction

Early history of LW study
Frequency-noise power spectral density

2. Intrinsic linewidth by spontaneous emissions

The Schawlow-Townes-Henry formula
Line narrowing by high photon flux at single-mode

3. Line broadening by thermal photons in THz QCLs

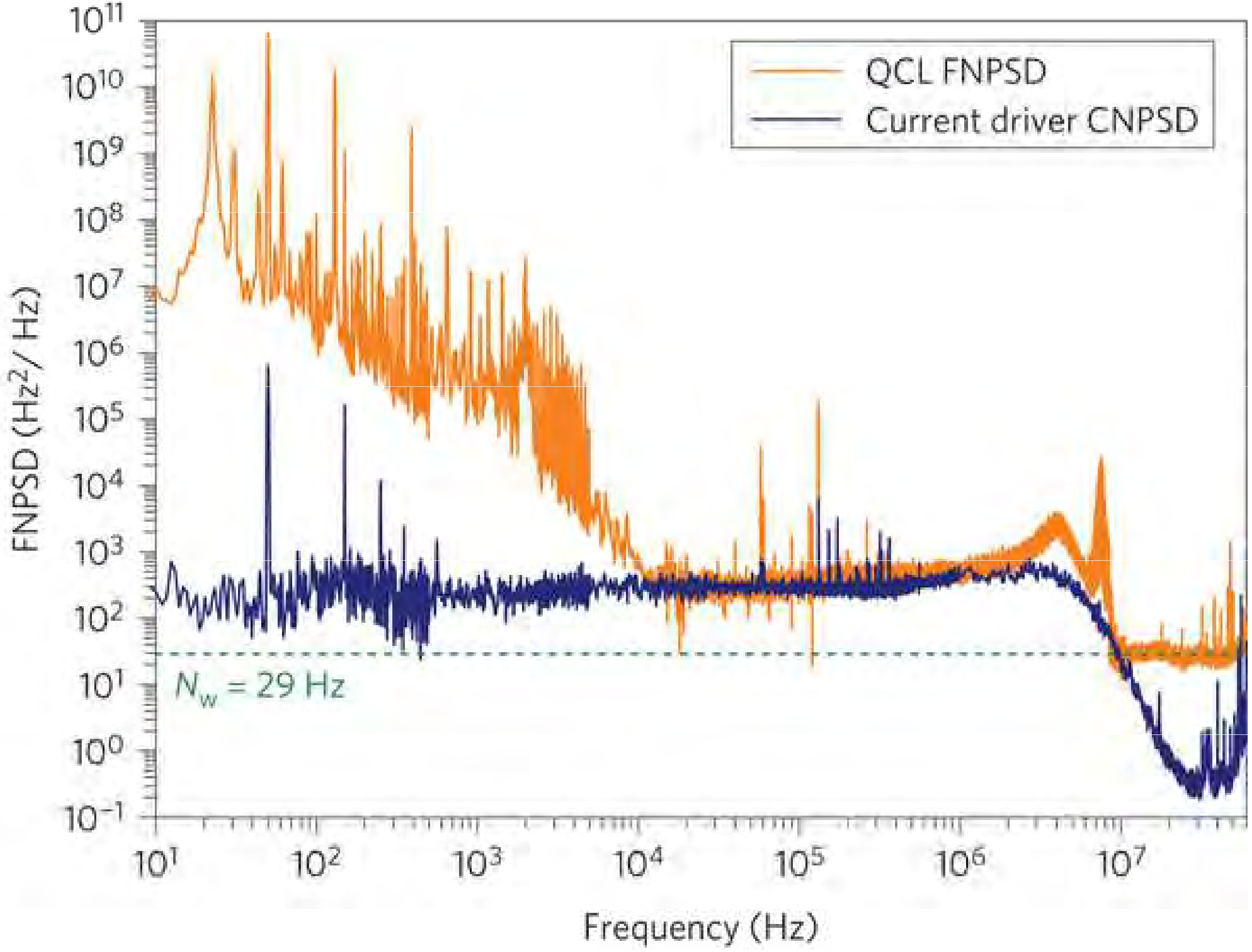
Generalized linewidth formula

4. Flicker frequency and electrical noises

How to suppress electrical flicker noise
Hot topic!

5. Summary and future

Frequency noise power spectral density of THz QCLs



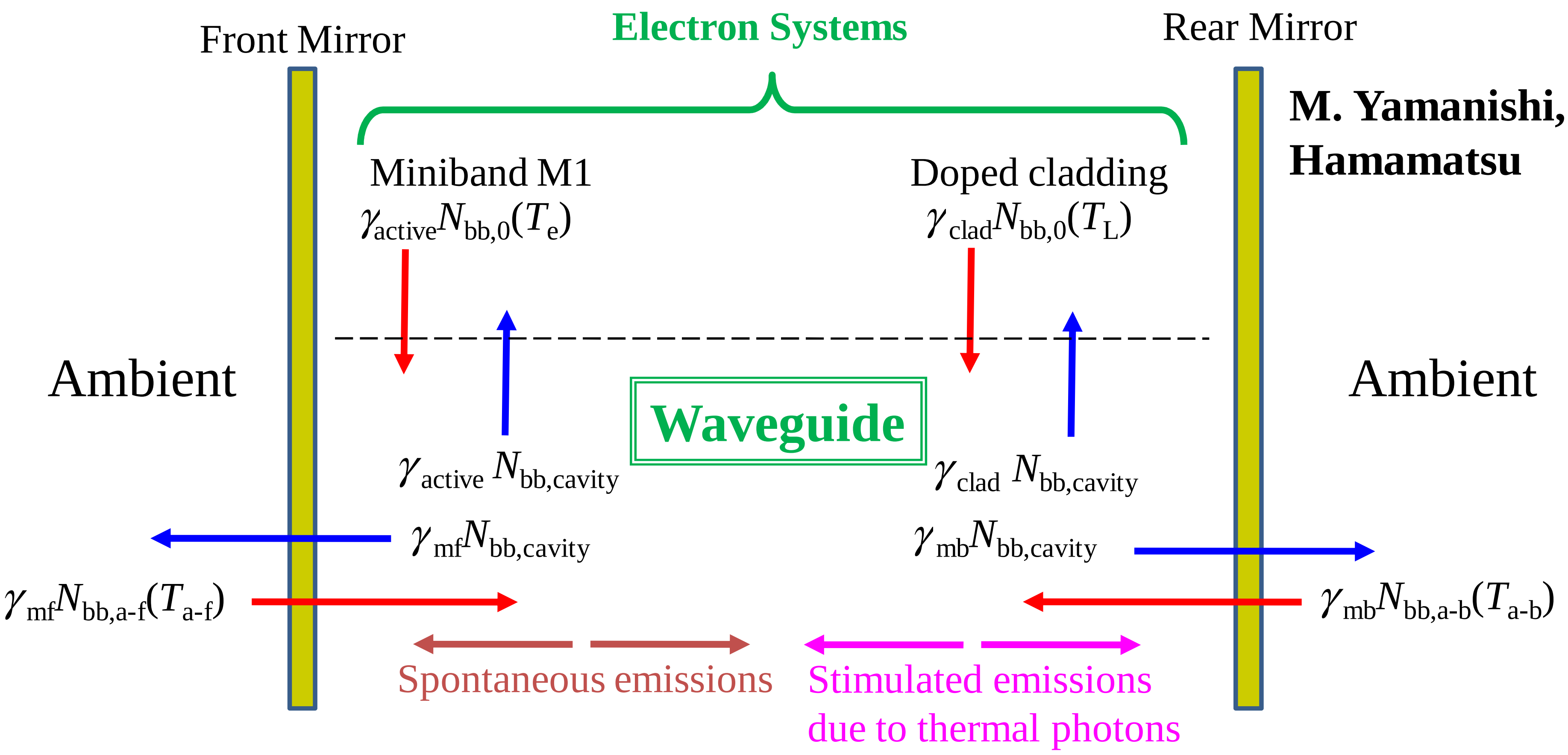
M. S. Vitiello et al. Nature Photon. **6**(8), 525-528 (2012).

TH=47.5 K

Intrinsic LW ~90 Hz, despite additional broadening due to thermal photons

Fluctuation and Dissipation Flows of Thermal Photons in a THz QCL

M. Yamanishi,
Hamamatsu



Front Mirror

Electron Systems

Rear Mirror

Miniband M1
$\gamma_{\text{active}} N_{\text{bb},0}(T_e)$

Doped cladding
$\gamma_{\text{clad}} N_{\text{bb},0}(T_L)$

Ambient

Waveguide

Ambient

$\gamma_{\text{active}}\, N_{\text{bb,cavity}}$

$\gamma_{\text{clad}}\, N_{\text{bb,cavity}}$

$\gamma_{\text{mf}} N_{\text{bb,cavity}}$

$\gamma_{\text{mb}} N_{\text{bb,cavity}}$

$\gamma_{\text{mf}} N_{\text{bb,a-f}}(T_{\text{a-f}})$

$\gamma_{\text{mb}} N_{\text{bb,a-b}}(T_{\text{a-b}})$

Spontaneous emissions

Stimulated emissions
due to thermal photons

Noise Sources for LW Broadening:

(1) spontaneous emissions, γn_{sp},
(2) noisy stimulated emissions due to thermal photons $\gamma n_{\text{sp}} N_{\text{bb,cavity}}$,
(3) incoming fluctuations from the electron systems, $\gamma_{\text{active}} N_{\text{bb},0}(T_e) + \gamma_{\text{clad}} N_{\text{bb},0}(T_L)$
(4) incoming fluctuations from the ambient systems $\gamma_{\text{mf}} N_{\text{bb,a-f}}(T_{\text{a-f}}) + \gamma_{\text{mb}} N_{\text{bb,a-b}}(T_{\text{a-b}})$

The population-inversion parameter $n_{\text{sp}} = N_3/(N_3 - N_2)_{\text{th}}$ $\quad \gamma n_{\text{sp}} = M\beta(N_3/\tau_r)$

Roles of spontaneous emissions, stimulated emissions due to thermal photons and incoming thermal photon fluctuations in frequency fluctuations

Comparison of counter parts

Vacuum field fluctuation (VFF) (Invisible)	Spontaneous emissions (Stimulated emissions due to VFF)
Fluctuating (incoming) thermal photons	Stimulated emissions due to thermal photons (not net rate)

Absorption/dissipation does not contribute to the frequency fluctuations because of no phase change and no interference.

The intrinsic LW (FWHM) of the Lorentzian line shape of a THz QCL:

$$\delta\nu = \left\{\frac{(1+\alpha_c{}^2)\gamma}{4\pi N_{\mathrm{photon}}}\right\}\Big\{n_{\mathrm{sp}} + (n_{\mathrm{sp}}+1)\Big[(\gamma_{\mathrm{active}}/\gamma)\mathrm{N}_{\mathrm{bb,0}}(T_{\mathrm{e}})$$
$$+(\gamma_{\mathrm{clad}}/\gamma)\mathrm{N}_{\mathrm{bb,0}}(T_{\mathrm{L}}) + (\gamma_{\mathrm{mf}}/\gamma)\mathrm{N}_{\mathrm{bb,a\text{-}f}}(T_{\mathrm{a\text{-}f}}) + (\gamma_{\mathrm{mb}}/\gamma)\mathrm{N}_{\mathrm{bb,a\text{-}b}}(T_{\mathrm{a\text{-}b}})\Big]\Big\}.$$

The generalized LW formula (in M. Yamanishi, Optics Express, **20,** 28465 (2012)) is applicable to any cases between the two extreme limits: the quantum limit, $\hbar\omega_{\mathrm{photon}} >> k_{\mathrm{B}}T$, and thermal limit, $\hbar\omega_{\mathrm{photon}} << k_{\mathrm{B}}T$.

$P_{\mathrm{int}}=(\hbar\omega_{\mathrm{photon}})\gamma N_{\mathrm{photon}} \sim 15.06$ mW, i.e., $P_{\mathrm{out}} \sim 3.8$ mW per facet (close to a detected power of 2.5 mW) $n_{\mathrm{sp}} \sim 1.22$, $\alpha_c \sim 0.35$, $\hbar\omega_{\mathrm{photon}} \sim 10.4$ meV, $\gamma \sim 7.3\times10^{10}$1/s, $\alpha_{\mathrm{mf}}+\alpha_{\mathrm{mb}} \sim 4.55$ cm^{-1}, $\alpha_{\mathrm{active}} \sim 1.9$ cm^{-1}, $\alpha_{\mathrm{clad}} \sim 2.6$ cm^{-1}, $\mathrm{N}_{\mathrm{bb,a\text{-}f}}=\mathrm{N}_{\mathrm{bb,a\text{-}b}}=\mathrm{N}_{\mathrm{bb}} \sim 0.0855$, $\mathrm{N}_{\mathrm{bb,0}}(T_{\mathrm{e}}=90\mathrm{K}) \sim 0.354$, $\mathrm{N}_{\mathrm{bb,0}}(T_{\mathrm{L}}=76\mathrm{K}) \sim 0.257$.

$\delta\nu$ ~86.6 Hz (Theory) close to 91.1 Hz±30 Hz(Experiment).

~35 % (22 Hz) broadening by thermal photons for $\delta\nu$~64.2 Hz

An extremely high photon population $N_{\mathrm{photon}} \sim 1.24\times10^8$ at a stable single-mode leads to substantial line narrowing.

How about line broadening by thermal photons at higher temperatures ~200 K, namely larger N_{bb} and larger n_{sp}?

Outline of Presentation

1. Introduction

Early history of LW study
Frequency-noise power spectral density

2. Intrinsic linewidth by spontaneous emissions

The Schawlow-Townes-Henry formula
Line narrowing by high photon flux at single-mode

3. Line broadening by thermal photons in THz QCLs

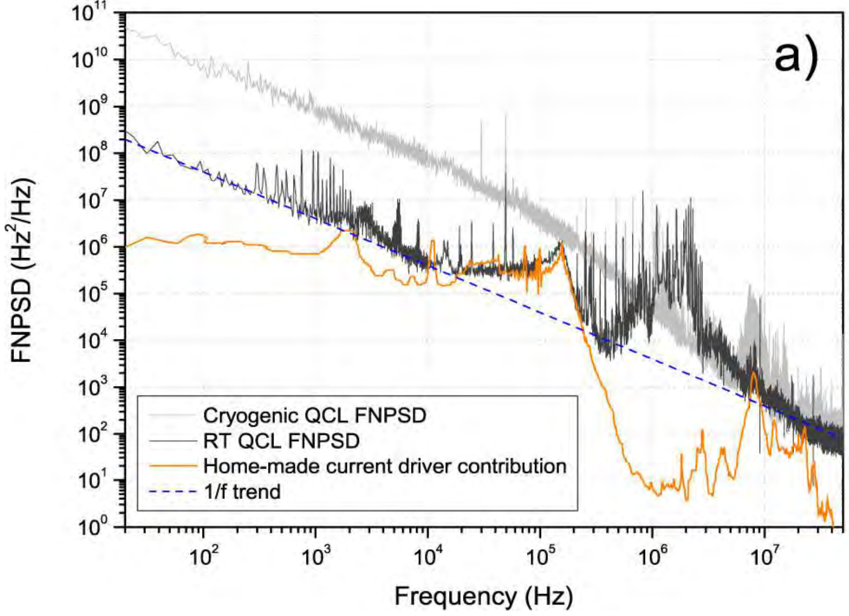
Generalized linewidth formula

4. Flicker frequency and electrical noises

How to suppress electrical flicker noise (by dopant positioning)
Hot topic!
For details, T. Hirohata et al., Poster paper, No. 11, IQCLSW 2014

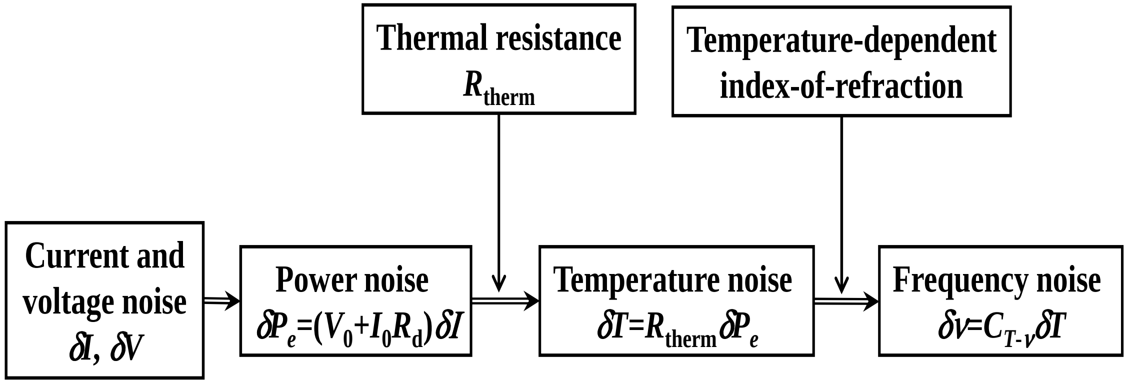
5. Summary and future

Why is suppression of flicker frequency-noise demanded?



S. Bartalini et al., Optics Express **19**, 17996 (2011). (in collaboration between INO and HPK)

- Flicker Frequency-Noise => Linewidth of 400 kHz (>>Intrinsic LW 260 Hz) in a free-running QCL at room temperature



Thermal resistance
R_{therm}

Temperature-dependent index-of-refraction

Current and voltage noise
δI, δV

Power noise
$\delta P_e=(V_0+I_0R_d)\delta I$

Temperature noise
$\delta T=R_{\text{therm}}\delta P_e$

Frequency noise
$\delta\nu=C_{T\text{-}\nu}\delta T$

Hypothesized by S. Borri et al. IEEE J-QE **47**, 984 (2011) and assured experimentally by L. Tombez et al. Optics Express **20**, 6851 (2012), CLEO 2013.

$S_{\delta I}(f_N)$----------------(Power-spectral density)-------------$S_{\delta\nu}(f_N)=[C_{T\text{-}\nu}R_{\text{therm}}(V_0+I_0R_d)]^2S_{\delta I}(f_N)$

- Suppression of electrical flicker noise (not by locking techniques)

cf I. Galli et al., “Comb-assisted subkilohertz linewidth quantum cascade laser for high-precision mid-ir spectroscopy” APL **102**, 121117 (2013) [INO+HPK] But, bulky optics are required!

Approximate (but well-validated) formula for LW in Gaussian-type line shape for the PSD, $S_{\delta\nu}(f_N) = N_{\text{flicker}}/f_N$, [Gianni Di Domenico, et al., Applied Optics **49**, 4801 (2010)]

$$\delta\nu = f_m \frac{8\ln 2}{\pi}[\ln(f_m T_w)]^{1/2} = (N_{\text{flicker}})^{1/2}\left(\frac{8\ln 2}{\pi}\right)^{1/2}[\ln(f_m T_w)]^{1/2},$$

where the characteristic frequency, $f_m = [\pi/2(2\ln 2)^{1/2}]\,(N_{\text{flicker}})^{1/2}$ and $T_w \sim 10$ ms: the time window.

We need very strong flicker noise suppression for a line-narrowing.

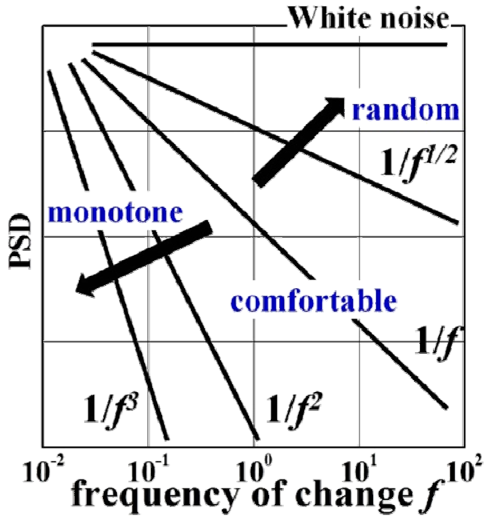
For instance, a line-narrowing down to $(1/10)\delta\nu$ is given by stronger noise suppression, $(1/100)N_{\text{flicker}}$.

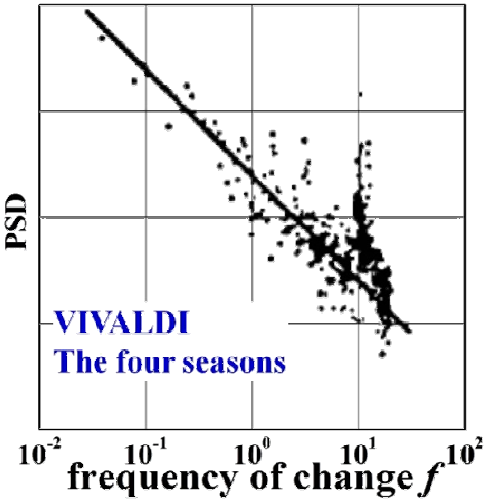
Very strong electrical flicker noise suppression is demanded!

The flicker fluctuation phenomenon globally existing in the Universe

Twinkling of star light, Orbit of star, Murmuring of stream, Pleasant music,

Neural pulse propagation, Heart beat of (healthy) human, Traffic flow in a high way, Information flow in a network system, Magnitude versus frequency of earth quake, Fluctuation associated with phase transition, Electrical flicker noise in electron devices, lasers, and detectors





Le Quattro Stagioni by Antonio Vivaldi

[from CyberWorkshop Essay]

The flicker fluctuations existing naturally are partly regulated (partly synergetic) and partly random, but their physical origins are still open to questions in the most of cases.

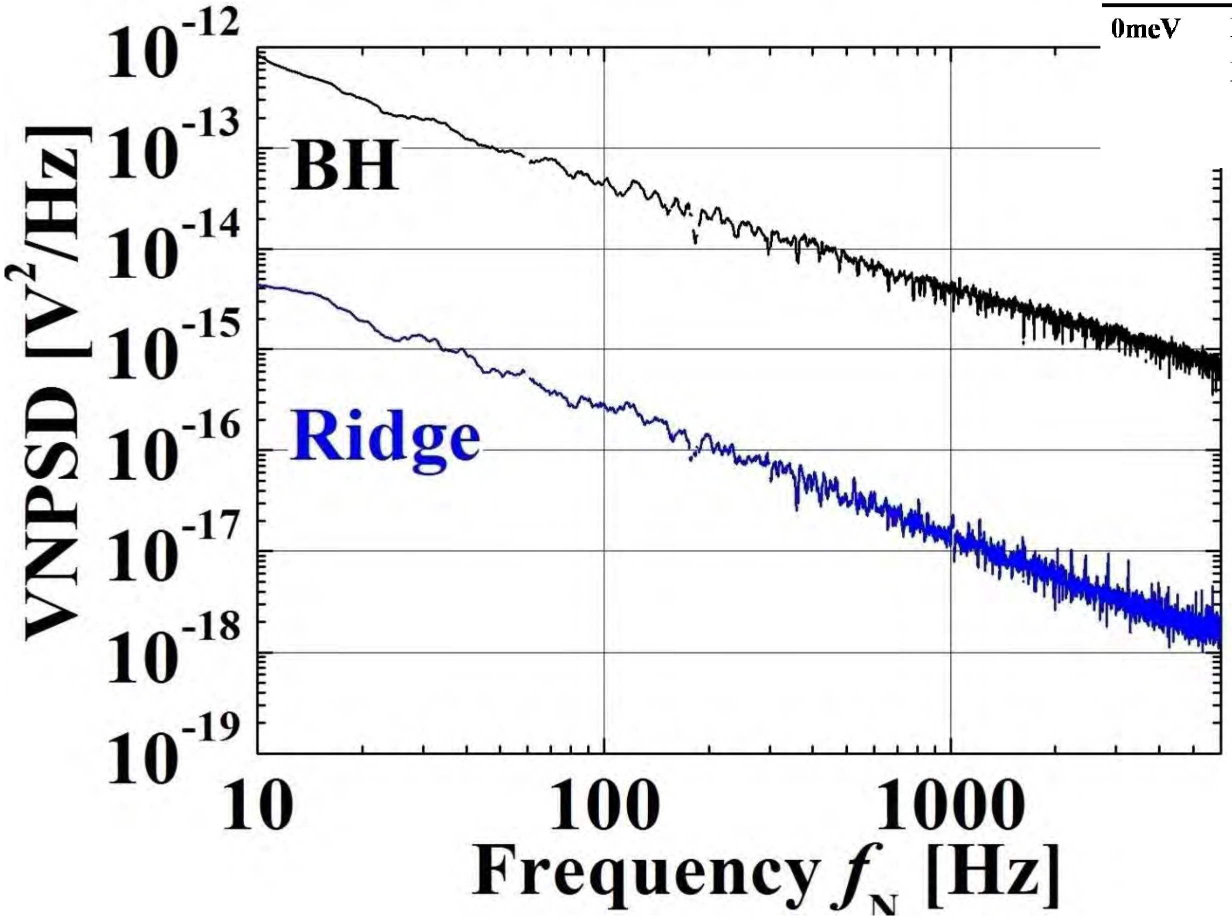
How about in semiconductors

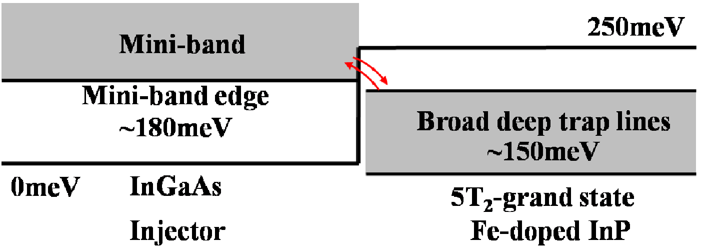
- **Physical origins? Long running debate over Number-fluctuation (1955) *versus* Mobility-fluctuation (1969) models. Which model is correct?**
- **Statistical stationary? Relation with fractal and scale invariance?**
- **The statement: "It is probably fair comment to say that to many physicist the subject of fluctuations (or "noise" to put it bluntly) appears rather esoteric and even pointless; spontaneous fluctuations seem nothing but an unwanted evil which only an unwise experimenter would encounter!"** [D. K. C. Macdonald: "Noise and fluctuations" (Wiley, Now York, 1962)]
- Flicker fluctuation has been untouchable so far!
- **"We have comparatively little knowledge about the microscopic origins of voltage fluctuations (even) in a simple resister."** [P. Dutta and P. M. Horn: "Low-frequency fluctuations in solid: 1/*f* noise," Rev. Modern Physics, vol. 53, No. 3 July 1981]
- Our **trial for manipulation of electrical flicker noise is a challenge to the Nature!**

For details, T. Hirohata et al., Poster paper, IQCLSW 2014 .

Comparison of noise levels in BH- and ridge-QCLs

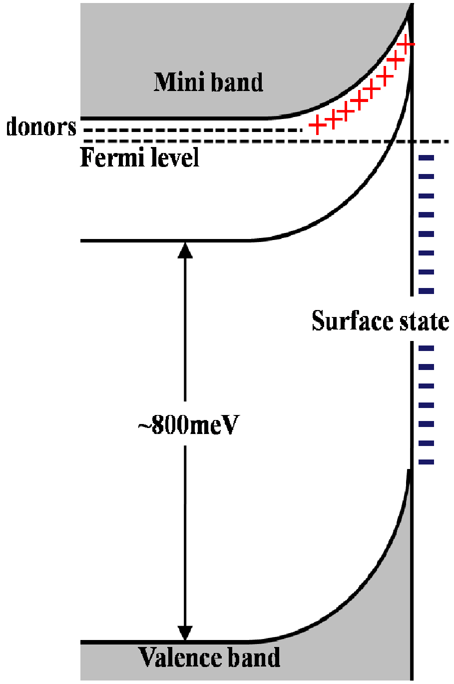
80 K





BH structure

D. C. Look, *Phys. Rev.* 20, 4160(1979).

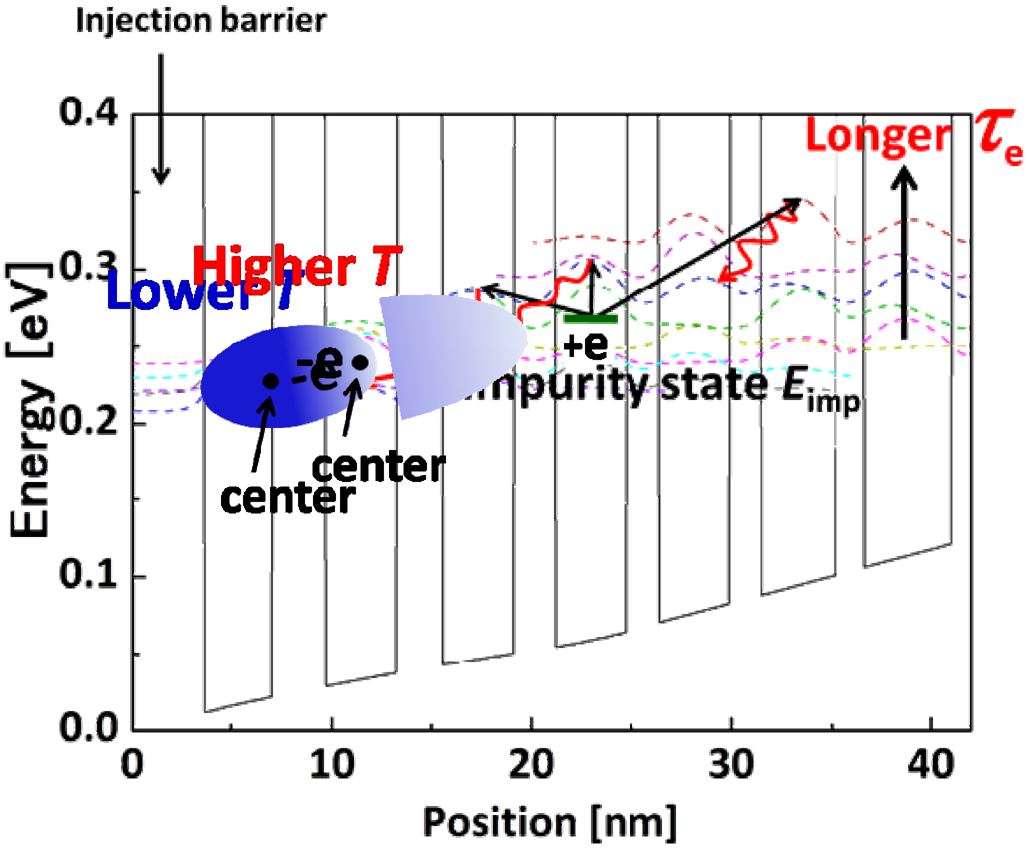


Ridge structure

VNPSDs of BH device are 20 dB higher than ridge device.

Similar to L. Tombez, CLEO 2013

Emission and capture model for voltage noise in a doped-injector



E_{imp} around corresponding subband
[G. Bastard, Phys. Rev. **B24** 4714 (1981)]

Dipole length

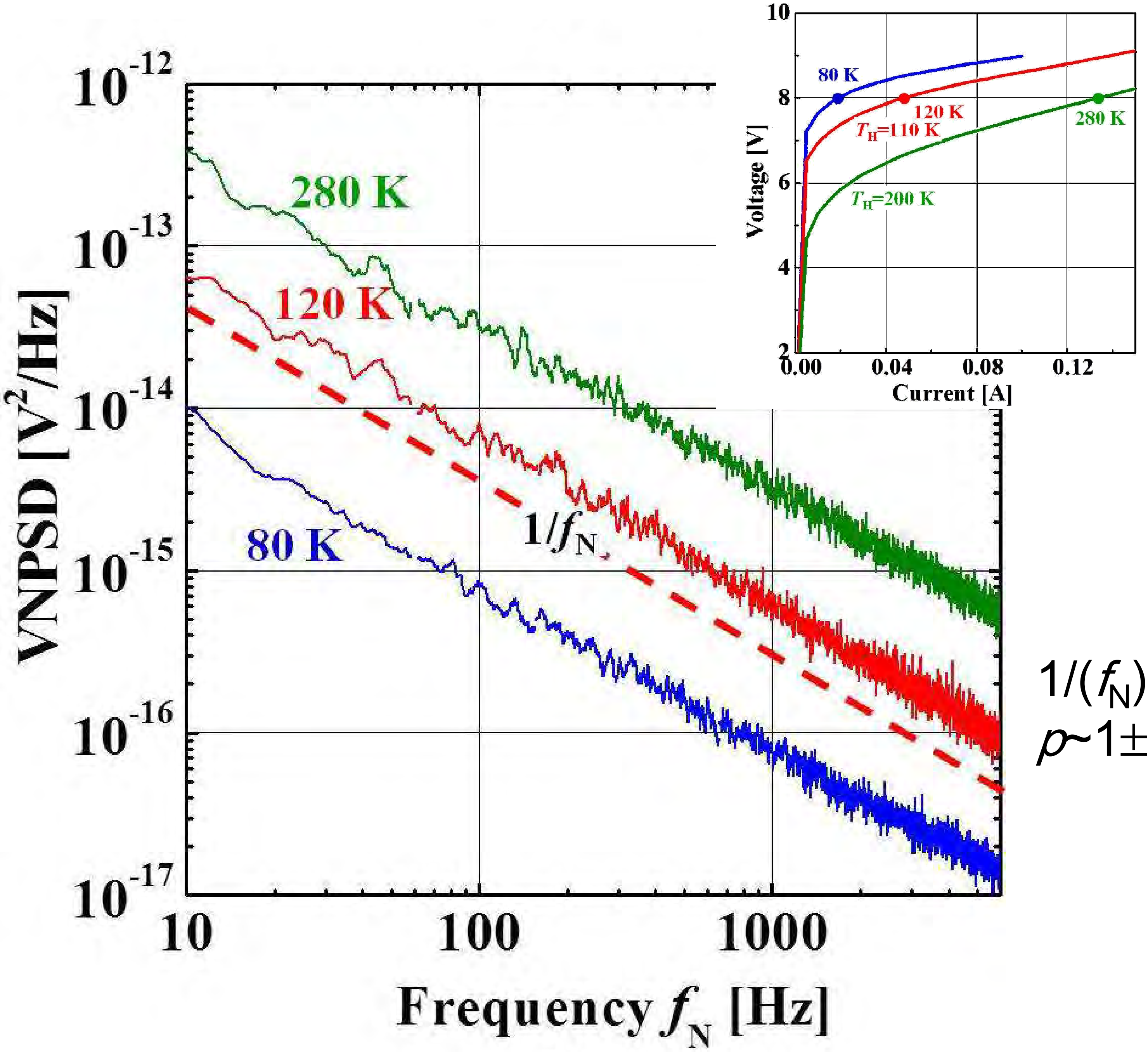
$$Z(z_{imp}) = \frac{\left| \sum_{i=1}^{I} \exp[-E_i / k_B T_e] \int dz (z - z_{imp}) \{\psi_i(z)\}^2 \right|}{\sum_{i=1}^{I} \exp[-E_i / k_B T_e]}$$

M=40, n_{imp}=1 × 10^{11} 1/cm^2,
S=14 µm × 1mm, ε_s=14,
τ_{eL}=100 ms, τ_{eS}=0.1 µs

Voltage-noise power spectral density

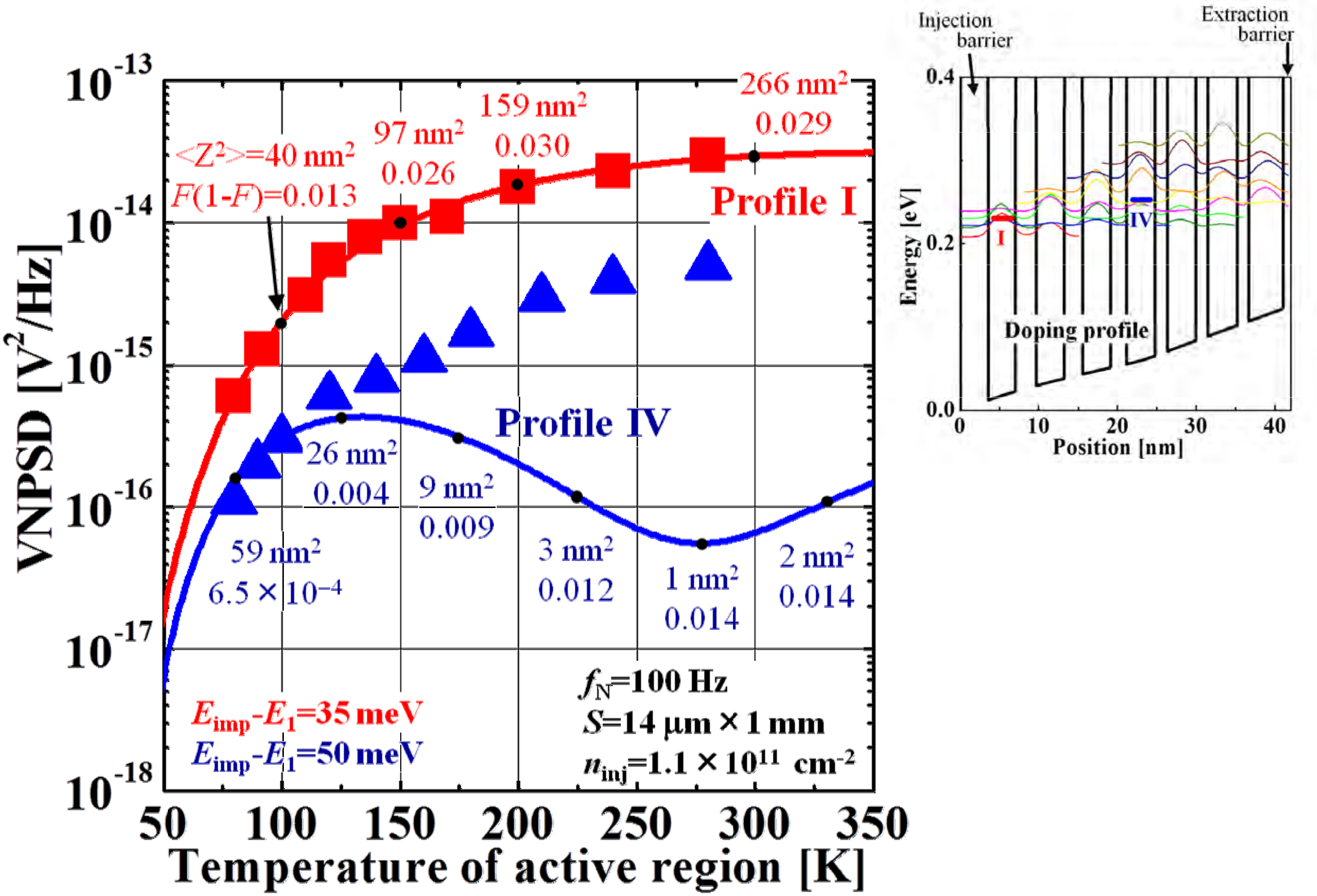
$$S_{\delta V}(f_N) = \frac{M n_{imp} S\, F(E_{imp}, E_F, T_e)[1 - F(E_{imp}, E_F, T_e)] \int (e Z(z_{imp}) / \varepsilon_0 \varepsilon_s S)^2 P_I(z_{imp}) dz_{imp}}{\ln(\tau_{eL} / \tau_{eS}) f_N}$$

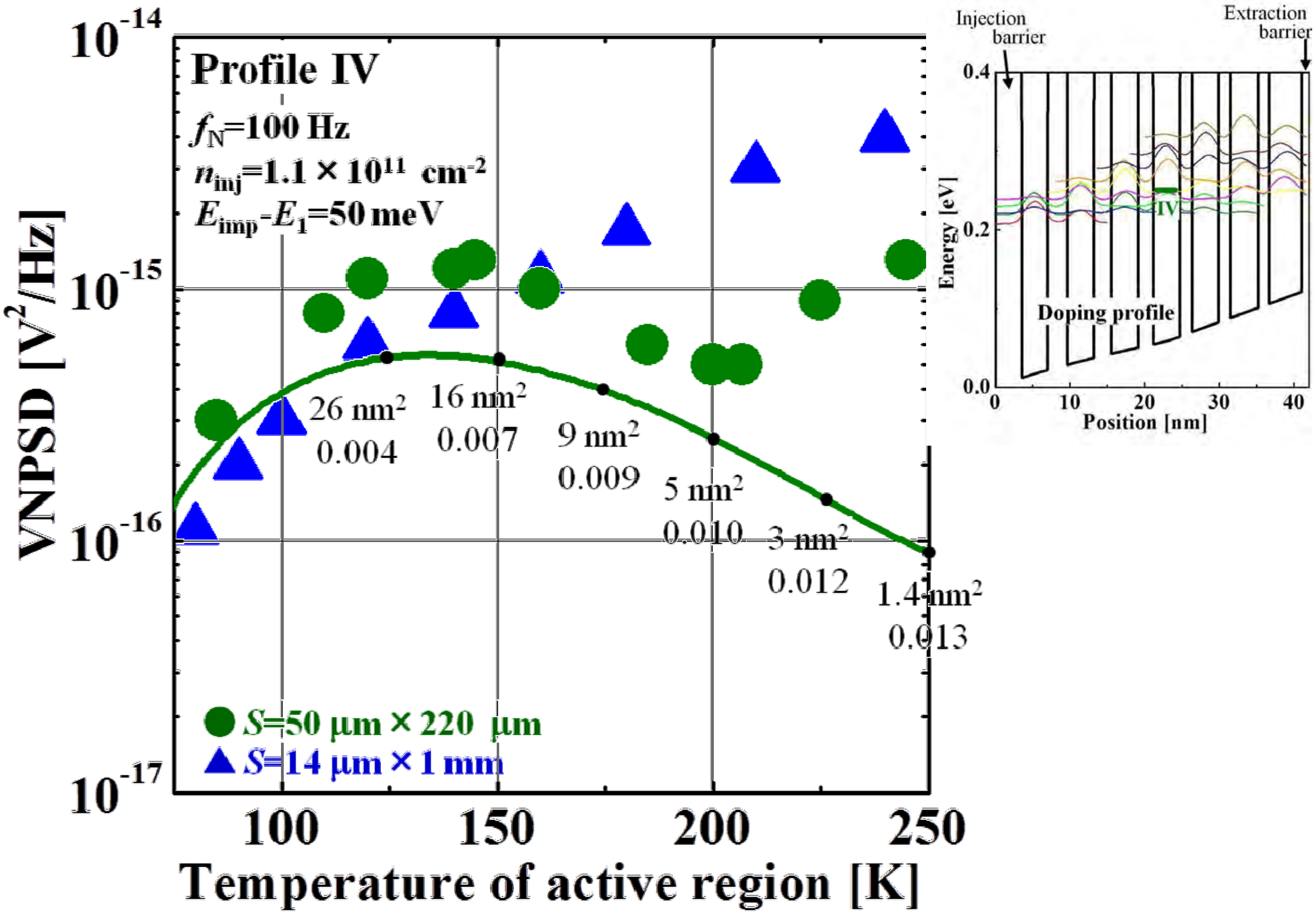
$P_I(z_{imp})$: the normalized distribution probability of impurities



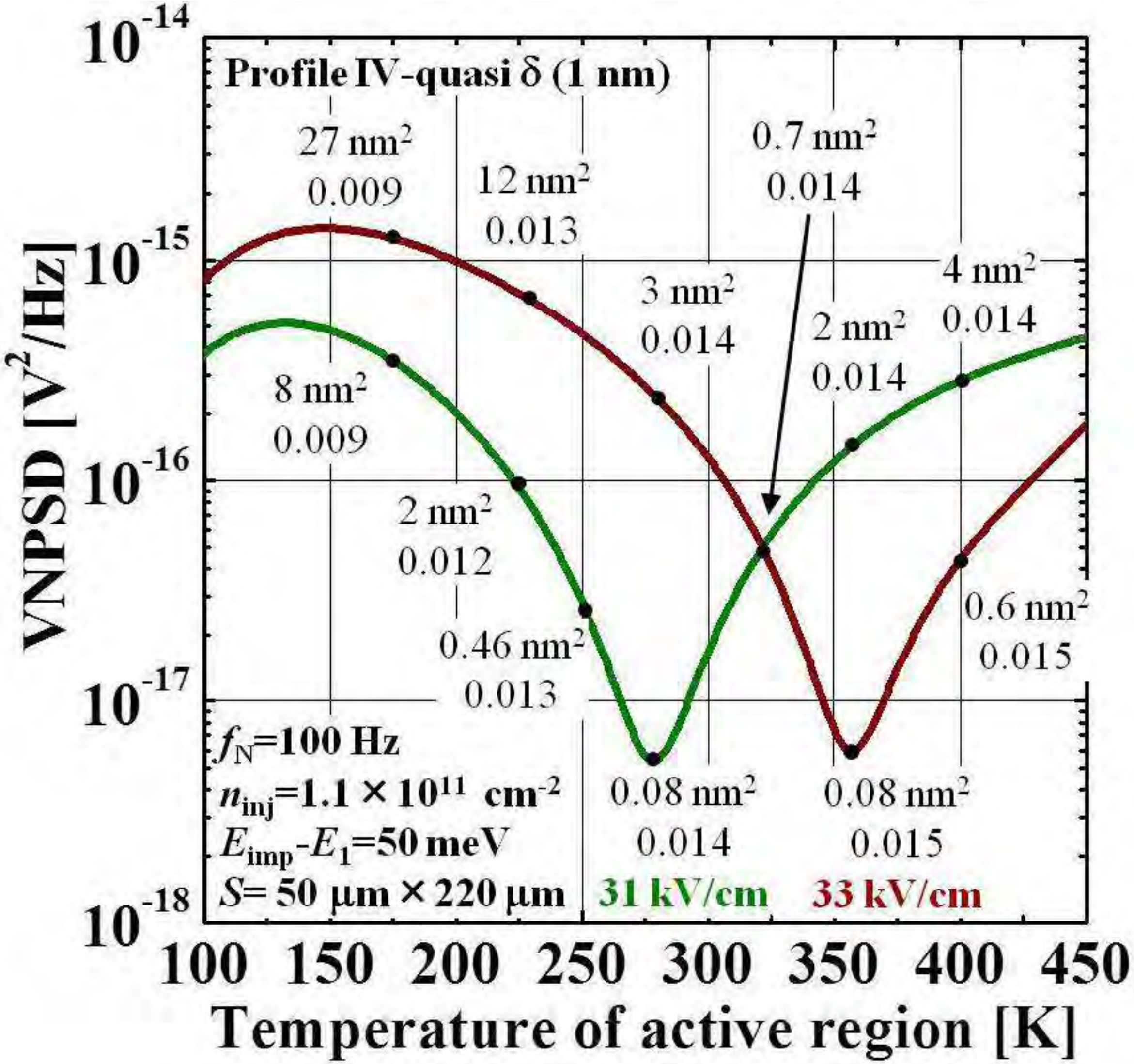
$1/(f_N)^p$ where

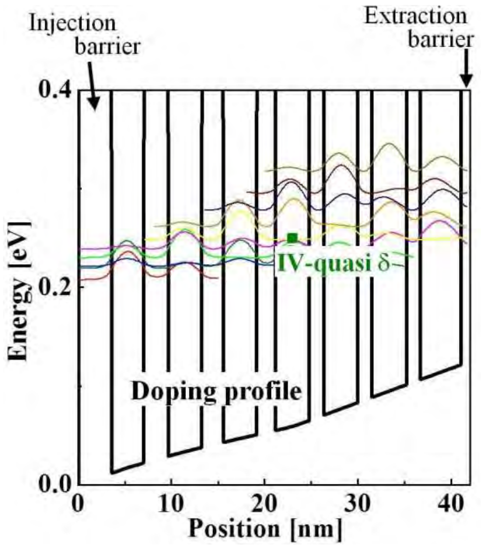
$p \sim 1 \pm 0.15$


VNPSD [V^2/Hz]
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
50
100
150
200
250
300
350
Temperature of active region [K]
$\langle Z^2 \rangle$=40 nm^2
$F(1-F)$=0.013
97 nm^2
0.026
159 nm^2
0.030
266 nm^2
0.029
Profile I
Profile IV
26 nm^2
0.004
9 nm^2
0.009
59 nm^2
6.5×10^{-4}
3 nm^2
0.012
1 nm^2
0.014
2 nm^2
0.014
E_{imp}-E_1=35 meV
E_{imp}-E_1=50 meV
f_N=100 Hz
S=14 µm × 1 mm
n_{inj}=1.1 × 10^{11} cm^{-2}
Injection barrier
Extraction barrier
0.4
0.2
0.0
Energy [eV]
I
IV
Doping profile
0
10
20
30
40
Position [nm]


Profile IV
f_N=100 Hz
n_{inj}=1.1 × 10^{11} cm^{-2}
E_{imp}-E_1=50 meV
26 nm^2
0.004
16 nm^2
0.007
9 nm^2
0.009
5 nm^2
0.010
3 nm^2
0.012
1.4 nm^2
0.013
● S=50 μm × 220 μm
▲ S=14 μm × 1 mm
VNPSD [V^2/Hz]
10^{-14}
10^{-15}
10^{-16}
10^{-17}
100
150
200
250
Temperature of active region [K]
Injection barrier
Extraction barrier
0.4
0.2
0.0
Energy [eV]
IV
Doping profile
0
10
20
30
40
Position [nm]

Noise suppression by quasi-delta doping





VNPSD 5×10^{-18} V^2/Hz at 100 Hz, i.e., CNPSD 6×10^{-20} A^2/Hz for R_d=15 ohm

→ Narrow LW of a free running QCL ~5 kHz for Tw=10 ms

(Inferred) CNPSD 10^{-17} A^2/Hz at 100 Hz and 100 K: L. Tombez et al. CLEO 2013

Summary and future

1. Introduction

   Frequency-noise power spectral density

2. Intrinsic linewidth by spontaneous emissions

   Line narrowing by high photon flux at single-mode

3. Line broadening by thermal photons in THz QCLs

   Generalized linewidth formula. Line-broadening at higher temperatures?

4. Flicker frequency and electrical noises: Hot topic!

   How to suppress electrical flicker noise by dopant positioning.
   (For details, T. Hirohata et al., Poster paper, No. 11, IQCLSW 2014)

5. (Near) Future

Removal of serious obstacles: surface states and deep traps
Understanding of microscopic physics underlying flicker noise
suppression involved in feedback and phase locking schemes
Linewidth of THz sources based on DFG scheme